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EARLE R. HEDRICK

DAVID R. CURTISS WILLIAM R. LONGLEY

WITH THE ASSISTANCE OF

HAROLD T. DAVIS HARRY W. KUHN

THEOPHIL H. HILDEBRANDT DERRICK N. LEHMER

JOHN R. KLINE HENRY L. RIETZ

CAROLINE E. SEELY

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THE APRIL MEETING IN NEW YORK

The three hundred fifth regular meeting of the American Mathematical Society was held at Columbia University on Friday and Saturday, April 14–15, 1933. The attendance included the following one hundred twenty-one members of the Society:

C. R. Adams, R. B. Adams, R. P. Agnew, R. L. Anderson, R. G. Archibald, Max Astrachan, M. J. Babb, N. H. Ball, M. F. Becker, A. H. Black, H. F. Bohnenblust, Samuel Borofsky, Joseph Bowden, A. B. Brown, J. H. Bushey, S. S. Cairns, R. H. Cameron, B. H. Camp, P. A. Caris, Alonzo Church, W. R. Church, J. A. Clarkson, J. M. Clarkson, W. B. Coleman, George Comenetz, E. L. Culbreth, E. H. Cutler, M. F. Deuring, B. F. Dostal, Jesse Douglas, Arnold Dresden, J. K. Dyer, H. T. Engstrom, J. M. Feld, W. B. Fite, D. A. Flanders, W. W. Flexner, R. M. Foster, Orrin Frink, C. A. Garabedian, H. M. Gehman, B. P. Gill, R. E. Gilman, H. S. Grant, M. C. Gray, C. C. Grove, R. M. Gut, Alan Hazeltine, Einar Hille, Lulu Hofmann, T. R. Hollcroft, G. M. Hopper, I. O. Horsfall, Harold Hotelling, R. S. Hoyt, J. C. Hughes, Nathan Jacobson, S. A. Joffe, R. A. Johnson, F. E. Johnston, Edward Kasner, J. R. Kline, P. A. Knedler, Harry Langman, V. V. Latshaw, Elihu Lazarus, Solomon Lefschetz, A. K. Leon, E. R. Lorch, E. O. McCormick, L. A. MacColl, H. M. MacNeille, A. E. Meder, H. H. Mitchell, Richard Morris, G. W. Mullins, C. A. Nelson, E. P. Northrop, Oystein Ore, Victor Perlo, H. R. Phalen, R. S. Pieters, Hillel Poritsky, R. G. Putnam, H. W. Raudenbush, M. S. Rees, J. F. Ritt, S. L. Robinson, Benjamin Rosenbaum, J. B. Rosser, I. J. Schoenberg, C. E. Seely, Casper Shanok, I. M. Sheffer, J. A. Shohat, C. G. Shover, L. G. Simons, Abraham Sinkov, M. H. Stone, E. E. Strock, H. G. Swain, Paul Szabo, J. D. Tamarkin, H. B. Thiessen, T. Y. Thomas, E. W. Titt, C. C. Torrance, C. B. Tucker, J. V. Uspensky, R. J. Walker, Louis Weisner, Albert Wertheimer, A. P. Wheeler, H. S. White, Hassler Whitney, C. R. Wilson, W. A. Wilson, Clement Winston, H. P. Wirth, R. C. Yates, Leo Zippin.

The meeting opened on Friday morning with two sectional sessions, Geometry and Algebra, and Analysis and Applications. Friday afternoon was devoted to an address by Professor Oystein Ore, on *Abstract ideal theory and applications*, with discussion led by Dr. M. F. Deuring and Professor H. T. Engstrom. A general session was held on Saturday morning, at the close of which Professor J. V. Uspensky delivered an address entitled *Application of number theory to some problems of integration of elementary functions*. The addresses of Professors Ore and Uspensky were delivered at the invitation of the program committee.

Greetings were sent by telegram to the members of the Society at Chicago on the occasion of the symposium held there in memory of the late Professor E. H. Moore, expressing the desire of the New York group to join in honoring that distinguished mathematician. These greetings were acknowledged by telegram by the Chicago group.

Titles and cross references to the papers (other than invited addresses) read at this meeting follow below; papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 14 were read before the section of Geometry and Algebra, Professor W. A. Wilson presiding; those numbered 15 to 29 before the section of Analysis and Applications, Professor J. D. Tamarkin presiding; those numbered 30 to 34 before the general session, Professor C. R. Adams presiding. Professor H. H. Mitchell presided during Professor Ore's address, and Professor J. D. Tamarkin during Professor Uspensky's. Mr. Claytor was introduced by Professor J. R. Kline, and Dr. Saks by Professor J. D. Tamarkin.

1. *Characteristics of multiple curves and their residuals*, by Professor T. R. Hollcroft. (Abstract No. 39–5–148.)

2. *Types of involutorial space transformations associated with certain rational curves. Composite basis elements*, by Dr. Amos Black. (Abstract No. 39–5–149.)

3. *Involutorial line transformations defined by Cremona plane involutions*, by Dr. J. M. Clarkson. (Abstract No. 39–5–150.)

4. *On the triangulation of regular loci*, by Dr. S. S. Cairns. (Abstract No. 39–5–151.)

5. *Concerning the definition of limit point*, by Professor H. M. Gehman. (Abstract No. 39–5–152.)

6. *A problem of Klein concerning the real roots of an algebraic equation*, by Dr. I. J. Schoenberg. (Abstract No. 39–5–153.)

7. *On the zeros of certain zeta functions*, by Dr. M. F. Deuring. (Abstract No. 39–5–154.)

8. *Convex polyhedra and criteria for irreducibility*. Preliminary report, by Mr. Casper Shanok. (Abstract No. 39–5–155.)

9. *Normal congruences of parabolas*, by Professor Edward Kasner. (Abstract No. 39–5–156.)

10. *Concerning powers of certain classes of ideals in a cyclotomic realm which give the principal class*, by Mr. H. S. Grant. (Abstract No. 39–5–157.)

11. *Topological immersion of Peanian continua in a spherical surface*, by Mr. W. S. Claytor. (Abstract No. 39–5–158.)

12. *Convex domains and linear combinations of continuous functions*, by Dr. I. J. Schoenberg. (Abstract No. 39–5–159–*t*.)

13. *Decompositions of continua by means of local separating points*, by Dr. G. T. Whyburn. (Abstract No. 39–5–160–*t*.)

14. *On a certain function connected with polynomials in a Galois field*, by Professor Leonard Carlitz. (Abstract No. 39–5–161–*t*.)

15. *New necessary and sufficient conditions that a transformation be almost periodic*, by Mr. R. H. Cameron. (Abstract No. 39–5–162.)

16. *Differentiable functions defined on closed sets*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 39–5–163.)

17. *Certain linear functional equations, and their relation to a regular singular point of a differential equation*, by Professor I. M. Sheffer. (Abstract No. 39–5–164.)

18. *On the numerators of the convergents of the Stieltjes continued fractions*, by Dr. Jacob Sherman. (Abstract No. 39–5–165–*t*.)

19. *On the mechanical quadratures formulas related to Hermite and Laguerre polynomials*, by Dr. Clement Winston. (Abstract No. 39–5–166–*t*.)

20. *Application of analytic function theory to two-dimensional elasticity problems*, by Dr. Hillel Poritsky. (Abstract No. 39–5–167.)

21. *The χ^2 test of goodness of fit*, by Dr. Selby Robinson (National Research Fellow). (Abstract No. 39–5–168.)

22. *On non-factorable transformations of double sequences*, by Professor C. R. Adams. (Abstract No. 39–5–169.)

23. *Integral functions obtained by compounding polynomials*, by Professor J. F. Ritt. (Abstract No. 39–5–170.)

24. *Application of Bernoulli polynomials of negative order to differencing*. Second paper, by Dr. B. F. Kimball. (Abstract No. 39–5–171–*t*.)

25. *On some applications of the Tchebycheff inequality for definite integrals*, by Professor J. A. Shohat. (Abstract No. 39–5–172–*t*.)

26. *A criterion for testing the hypothesis of mutual independence of a set of normal multivariate populations*, by Dr. S. S. Wilks (International Research Fellow). (Abstract No. 39–5–173–*t*.)

27. *Line integrals as a measure of polynomial approximation to analytic functions*, by Professor J. L. Walsh. (Abstract No. 39–5–174–*t*.)

28. *Addition to the note "On some functionals,"* by Dr. Stanislaw Saks. (Abstract No. 39–5–175–*t*.)

29. *On equations in mixed differences*, IV, by Mr. L. B. Robinson. (Abstract No. 39–5–176–*t*.)

30. *Derivatives with respect to functions of bounded variation*, by Professor R. L. Jeffery. (Abstract No. 39–5–177–*t*.)

31. *On definitions of bounded variation for functions of two variables*, by Professor C. R. Adams and Mr. J. A. Clarkson. (Abstract No. 39–5–178.)

32. *On the transfinite diameter for a linear interval*, by Professor J. A. Shohat. (Abstract No. 39–5–179.)

33. *Questions of relative inclusion in the domain of Hausdorff matrices*, by Professors Einar Hille and J. D. Tamarkin. (Abstract No. 39–5–180.)

34. *On the summability of Fourier series*, IV, by Professors Einar Hille and J. D. Tamarkin. (Abstract No. 39–5–181.)

J. R. KLINE, *Associate Secretary*

THE APRIL MEETING IN CHICAGO

The three hundred sixth regular meeting of the Society was held at the University of Chicago on Friday and Saturday, April 14–15, 1933. About one hundred twenty persons attended the meetings, among whom were the following ninety members of the Society:

A. A. Albert, R. P. Baker, R. H. Bardell, R. W. Barnard, Walter Bartky, E. F. Beckenbach, M. M. Beenken, S. F. Bibb, G. D. Birkhoff, G. A. Bliss, Henry Blumberg, J. W. Bower, H. R. Brahana, L. E. Bush, W. D. Cairns, R. D. Carmichael, J. E. Case, E. W. Chittenden, A. B. Coble, R. P. Conkling, Max Coral, Byron Cosby, D. R. Curtiss, L. E. Dickson, Arnold Emch, H. S. Everett, Cornelius Gouwens, C. H. Graves, L. M. Graves, L. W. Griffiths, W. W. Hart, M. L. Hartung, M. R. Hestenes, E. H. C. Hildebrandt, T. H. Hildebrandt, J. J. L. Hinrichsen, T. F. Holgate, M. H. Ingraham, Dunham Jackson, J. M. Kinney, W. C. Krathwohl, C. C. Krieger, A. C. Ladner, E. P. Lane, R. E. Langer, M. I. Logsdon, E. B. Lytle, C. C. MacDuffee, W. D. MacMillan, H. F. MacNeish, Wilhelm Maier, Morris Marden, William Markowitz, A. E. May, J. R. Mayor, C. N. Moore, T. W. Moore, E. J. Moulton, Rufus Oldenburger, Gordon Pall, G. A. Parkinson, E. D. Pepper, H. P. Pettit, E. C. Pixley, H. H. Pixley, Tibor Radó, W. T. Reid, R. G. D. Richardson, H. L. Rietz, W. H. Roever, W. E. Roth, R. G. Sanger, Henry Schultz, E. B. Skinner, H. E. Slaught, Burke Smith, A. A. Stafford, R. C. Stephens, V. B. Teach, H. P. Thielman, W. J. Trjitzinsky, P. L. Trump, H. L. Turrittin, E. B. Van Vleck, Oswald Veblen, K. W. Wegner, F. B. Wiley, K. P. Williams, Y. K. Wong, F. E. Wood.

A meeting of the Council was held on Friday afternoon in Room 316, Eckhart Hall. The Secretary announced the election of the following persons to membership in the Society:

Professor George Allen Baker, Shurtleff College;
Professor Willard Alger Ballou, Junior College of Connecticut;
Mr. William Gerard Banks, Jr., Centenary College;
Mr. Louis Baron, Washington Square College, New York University;
Mr. Robert Eugene Basye, University of Texas;
Mr. Paul Anthony Benitz, Hunter College;
Professor William Beverley, Lafayette College;
Mr. Ernest Arthur Birkinshaw, Brandon College;
Professor Paul Boeder, Susquehanna University;
Professor Fannie W. Boyce, Wheaton College;
Mr. Carl Benjamin Boyer, Brooklyn College;
Mr. A. Day Bradley, Hunter College;
Miss Georgia Alberta Lee Caldwell, Spelman College;

Mr. Irval W. Carter, Stanford University;
Rev. James Edward Case, St. Louis University;
Sister Catherine Marie, College of Mount St. Vincent;
Mr. Si-Ping Cheo, University of Michigan;
Mr. Walter Barton Coleman, Lehigh University;
Dean Samuel Henry Cook, Berry College;
Professor Mary Jane Cox, Hollins College;
Mr. Laurence Lolin Cruise, Rochester Junior College;
Professor Walter Hampton Dabney, Alcorn Agricultural and Mechanical College;
Professor Anthony Davidson, Wilberforce University;
Professor Charlotte Isabel Davison, Wilson College;
Mr. Daniel B. DeLury, University of Saskatchewan;
Mr. Reginald Horton Downing, West Virginia University;
Professor Martin Linnaeus Drum, Bucknell University;
Professor Janet C. Durand, Beaver College;
Professor Charles R. Eason, Shaw University;
Dr. Robert Lewis Echols, College of the City of New York;
Professor G. Dan Estes, Arkansas State Teachers College, Conway;
Sister Esther Maria Kenna, College of St. Elizabeth;
Mr. Nicholas Fattu, Itasca Junior College;
Mr. James Sutherland Frame, Harvard University;
Mr. Alan Stuart Galbraith, Harvard University;
Professor Theodore Whidden Gibson, Long Island University;
Professor Thomas Edward Gilmer, Hampden-Sydney College;
Professor Jekuthiel Ginsburg, Yeshiva College;
Mr. James Bonaparte Greeley, 2d, Polytechnic Institute of Brooklyn;
Dr. Joseph A. Greenwood, Duke University;
Mr. Samuel L. Greitzer, Bronx, New York City;
Mr. Morris Halperin, Brooklyn, N. Y.;
Professor Olan Harvey Hamilton, San Antonio Junior College;
Mr. Charles Henry Harrington, Glendale Junior College;
Professor Anthony Keith Hinds, Western Carolina Teachers College;
Professor George Maceo Jones, Howard University;
Mr. John Dennis Keyes, Montana School of Mines;
Mr. Earl James Kilcullen, St. Joseph's College;
Dr. Edmund Chester Klipple, University of Texas;
Dr. Cypra Cecilia Krieger, University of Toronto;
Professor Cornelius Lanczos, Purdue University;
Mr. Elihu Lazarus, New York City;
Miss Alice Katharine Leon, Adelphi College;
Mr. Louis Augustine McCoy, Roxbury Memorial High School for Boys;
Professor Marshall Jefferson McNeal, Randolph-Macon College;
Mr. Julian Dassy Mancill, University of Alabama;
Mother Marie Kernaghan, Maryville College;
Professor Malcolm Ray Marsh, College of Mines and Metallurgy, El Paso, Texas;
Sister Mary Charlotte Fowler, Nazareth Junior College;

Sister Mary Domitilla Thuener, Villa Madonna College;
Sister Mary Nicholas Arnoldy, Marymount College;
Mr. Edward L. Mayo, Joliet High School and Junior College;
Professor Eugene R. Mittinger, John Carroll University;
Mr. Sidney Neuman, New York City;
Mr. Thomas Nicholson, Brooklyn College;
Professor Robert Earl Norris, Milwaukee State Teachers College;
Professor Nicholas M. Oboukhoff, Oklahoma Agricultural and Mechanical College;
Mr. Nathan Grier Parke, III, Princeton;
Professor Swen Franklin Parson, Northern Illinois State Teachers College;
Mr. Arthur Sylvester Peters, New York University;
Professor Maximilian Philip, College of the City of New York;
Mr. Richard S. Pieters, Princeton University;
Professor Zareh M. Pirenian, University of Florida;
Dr. Francis Regan, St. Louis University;
Professor William Henry Robinson, Joseph K. Brick Junior College;
Professor Lucile LeeOlene Rorex, Mountain Home College;
Mr. Clarence Ross, Junior College, Yakima, Wash.;
Mr. John Barkley Rosser, Princeton University;
Mr. Norman Norris Royall, Jr., Georgia School of Technology;
Mr. Henry Scheffé, University of Wisconsin;
Professor Raymond Francis Schnepp, St. Mary's University of San Antonio;
Mr. George Earl Schweigert, Johns Hopkins University;
Major Charles H. Sheffield, Marion Institute;
Professor George Llewellyn Shue, Montana School of Mines;
Professor Paul Ira Speicher, Albright College;
Mr. George Walter Streator, Bennett College for Women;
Professor Philip Mayfield Sunday, Jr., Jarvis Christian College;
Professor William D. Tallman, Montana State College;
Dean George Leward Washington, Technical College, Agricultural and Technical College, Greensboro, N. C.;
Professor Martha North Watson, Greenville Woman's College;
Mr. Howard B. Waxwood, Jr., Straight College;
Professor Eugene Paul Wigner, Institut für theoretische Physik der Technischen Hochschule, Berlin, Germany;
Mr. Jack Wolfe, Brooklyn College;
Professor Harvey Austice Wright, Transylvania College;
Professor Cyprian Yahner, St. Vincent College;
Miss Ruth Lu Tet Yap, University of Hawaii;

As Nominees of the University of Michigan:

Mr. Bassford C. Getchell, Ann Arbor;
Mr. Thomas N. E. Greville, Ann Arbor;
Mr. Kenneth B. Leisenring, Ann Arbor;
Mr. Franklin C. Smith, Ann Arbor;

As Nominees of the University of Pennsylvania:

Mr. Bernard B. August, Philadelphia;
Mr. Richard Pennington Bailey, University of Pennsylvania;
Mr. Irwin Biser, Philadelphia;
Mr. Philip Maclasky, Drexel Institute.

The Secretary announced that the following persons had entered the Society under a reciprocity agreement:

Professor Corradino Mineo, University of Palermo, Palermo, Italy;
Professor Leonida Tonelli, University of Pisa, Pisa, Italy.

At the meeting of the Council held on April 14, it was decided to omit the Christmas meeting in Chicago for 1933. The date of the Thanksgiving meeting in Cincinnati was set for December 1–2, 1933, and that of the 1934 spring meeting in Chicago for April 6–7, 1934.

It was announced that the Committee on the Bôcher Prize consists of Professors G. D. Birkhoff, chairman, R. D. Carmichael, and J. D. Tamarkin, and that President Coble has chosen to act with himself as Honorary Committee on Entertainment of Guests at the Summer Meeting, Professors D. R. Curtiss and E. B. Van Vleck together with the consuls of Hungary and Italy resident in Chicago.

The Editorial Committee of the Colloquium Publications reported that it has accepted a volume by Professor J. H. M. Wedderburn on *The Theory of Matrices*.

A committee was appointed consisting of Professors R. W. Barnard, E. W. Chittenden, L. M. Graves, T. H. Hildebrandt, and M. H. Ingraham to bring out the work of the late Professor E. H. Moore on *General Analysis*; it is expected that the committee will issue this in sections, one section to appear shortly.

On Friday afternoon there was a meeting in memory of Professor E. H. Moore at which the following of Professor Moore's former students addressed the Society:

Professor G. A. Bliss, *E. H. Moore—a biographical sketch*;
Professor L. E. Dickson, *Professor Moore's work in theory of groups and algebra*;
Professor Oswald Veblen, *Professor Moore's work on foundations*;
Professor E. W. Chittenden, *The introduction to general analysis*;

Professor T. H. Hildebrandt, *General analysis and integral equations*;

Professor M. H. Ingraham, *The algebraic ground work for general analysis*;

Professor R. W. Barnard, *The second general analysis theory of E. H. Moore.*

Mrs. Moore was in attendance at this meeting.

On Saturday morning Professor C. C. MacDuffee delivered a Symposium Lecture on *Matrices with elements in a principal ideal ring.*

On Friday evening members and their guests attended a dinner at Judson Court. President A. B. Coble was toastmaster. Secretary R. G. D. Richardson spoke of the present state of the Society. Professor H. S. Everett described plans for the Summer Meeting. Professor Felix Bernstein gave a short talk in which he compared E. H. Moore with Klein and Hilbert. In the concluding address Professor Enrico Bompiani expressed his pleasure at being in Chicago again. Ninety-five people attended this dinner.

Professors G. A. Bliss and Dunham Jackson presided at the Friday morning session, and President A. B. Coble presided on Friday afternoon, Saturday morning, and Saturday afternoon.

Titles and cross-references to the papers (other than invited addresses) read at this meeting follow below; papers whose abstract numbers are followed by the letter *t* were read by title. Professor Collins was introduced by Professor Ingraham, Mr. Steenrod by Professor R. L. Wilder, Mr. Nathan by Professor I. A. Barnett, Dr. Akeley by Professor Maier, and Mr. Alden by Professor F. R. Bamforth.

1. *On the isoperimetric inequality*, by Professor Tibor Radó and Dr. E. F. Beckenbach. (Abstract No. 39–3–114.)

2. *Geometry of two-component spinors*, by Professor Oswald Veblen. (Abstract No. 39–5–125.)

3. *A note on the problem of Bolza in the calculus of variations*, by Professor G. A. Bliss and Dr. M. R. Hestenes. (Abstract No. 39–5–123.)

4. *An inverse problem in differential equations*, by Professor R. E. Langer. (Abstract No. 39–5–126.)

5. *On a resolution of linear differential systems*, by Professor

I. S. Sokolnikoff and Dr. E. S. Sokolnikoff. (Abstract No. 39–5–127–*t*.)

6. *On the invariance of a generalized Gramian under the group of linear functional transformations of the third kind*, by Dr. H. P. Thielman. (Abstract No. 39–5–128.)

7. *Note on* (1, 2) *algebraic correspondences*, by Professor Theodore Bennett. (Abstract No. 39–5–129–*t*.)

8. *On an involutorial Cremona transformation in* S_r, by Professor Arnold Emch. (Abstract No. 39–5–130.)

9. *A generalization of the Steiner and Veronese surfaces*, by Mr. J. R. Mayor. (Abstract No. 39–5–132.)

10. *Sufficient conditions for the problem of Lagrange in the calculus of variations*, by Dr. M. R. Hestenes. (Abstract No. 39–5–124.)

11. *Symmetric monoids and cones of higher order in* S_3, by Professor Arnold Emch. (Abstract No. 39–5–131–*t*.)

12. *Contributions to the theory of transformations of nets in a space* S_n, by Professor V. G. Grove. (Abstract No. 39–5–133–*t*.)

13. *An unused system of coordinates in analytic geometry*, by Professor J. V. Collins. (Abstract No. 39–3–115–*t*.)

14. *Finite arc-sums*, by Mr. N. E. Steenrod. (Abstract No. 39–3–122–*t*.)

15. *The topology of* (*path*) *surfaces*, by Dr. C. B. Morrey, Jr. (National Research Fellow). (Abstract No. 39–5–134–*t*.)

16. *An analytic characterization of surfaces of finite Lebesgue area*, (II), by Dr. C. B. Morrey, Jr. (National Research Fellow). (Abstract No. 39–5–135–*t*.)

17. *Applications of the automorphs of sums of three squares to regularity of ternary quadratic forms*, by Dr. Gordon Pall. (Abstract No. 39–5–136.)

18. *On the relations between sums of three and four squares*, by Dr. Gordon Pall. (Abstract No. 39–5–137.)

19. *On the rational solutions of the matric equations* $P(X) = A$, by Professor M. H. Ingraham. (Abstract No. 39–3–116.)

20. *On universal sets of positive ternary quadratic forms*, by Professor A. A. Albert. (Abstract No. 39–5–139.)

21. *The convergence of series of orthogonal trigonometric sums*, by Professor Dunham Jackson. (Abstract No. 39–5–141.)

22. *On criteria for Fourier constants of L integrable functions*, by Professor C. N. Moore. (Abstract No. 39–5–142.)

23. *The existence of an extremum in problems of Mayer*, by Professor L. M. Graves. (Abstract No. 39–3–119.)

24. *Lamé's functions and Lucas' numbers*, by Professor Wilhelm Maier. (Abstract No. 39–5–143.)

25. *Further mean-value theorems*, by Professor Morris Marden. (Abstract No. 39–5–144.)

26. *Regular infinitesimal transformations in a composite function space*, by Mr. D. S. Nathan. (Abstract No. 39–3–120.)

27. *An invariantive characterization of Riemannian geometries*, by Dr. E. S. Akeley. (Abstract No. 39–5–147.)

28. *Towards a unification of the arithmetic of quadratic forms*, by Dr. Gordon Pall. (Abstract No. 39–5–138–*t*.)

29. *A note on the Dickson theorem on universal ternaries*, by Professor A. A. Albert. (Abstract No. 39–5–140–*t*.)

30. *The application of the theory of admissible numbers to time series with constant probability*, by Dr. Francis Regan. (Abstract No. 39–3–118–*t*.)

31. *On continued fractions which represent meromorphic functions*, by Professor H. S. Wall. (Abstract No. 39–5–145–*t*.)

32. *Expansions of two arbitrary analytic functions in series of rational functions*, by Dr. P. W. Ketchum. (Abstract No. 39–5–146–*t*.)

33. *Solution of* $f(x, y)\partial z/\partial x+g(x, y)\partial z/\partial y=0$ *in a neighborhood of a singular point*, by Mr. H. H. Alden. (Abstract No. 39–3–117–*t*.)

34. *Linear transformations in a composite function space*, by Mr. D. S. Nathan. (Abstract No. 39–3–121–*t*.)

M. H. INGRAHAM, *Associate Secretary*

R. E. A. C. PALEY—IN MEMORIAM

JAN. 7, 1907–APRIL 7, 1933

Raymond Edward Alan Christopher Paley, Fellow of Trinity College, Cambridge, and International Research Fellow at the Massachusetts Institute of Technology and at Harvard University, was killed by an avalanche on April 7, 1933, while skiing in the vicinity of Banff, Alberta. Although only twenty-six years of age, he was already recognized as the ablest of the group of young English mathematicians who have been inspired by the genius of G. H. Hardy and J. E. Littlewood. In a group notable for its brilliant technique, no one had developed this technique to a higher degree than Paley. Nevertheless he should not be thought of primarily as a technician, for with this ability he combined creative power of the first order. As he himself was wont to say, technique without "rugger tactics" will not get one far, and these rugger tactics he practiced to a degree that was characteristic of his forthright and vigorous nature.

Possessed of an extraordinary capacity for making friends and for scientific collaboration, Paley believed that the inspiration of continual interchange of ideas stimulates each collaborator to accomplish more than he would alone. Only the exceptional man works well with a partner, but Paley had collaborated successfully with many, including Littlewood, Pólya, Zygmund, and Wiener.

From Eton Paley went to Trinity where he achieved high distinction and was graduated with every honor open to a mathematical student. As a pupil of and collaborator with Littlewood he engaged in most interesting investigations of Fourier series; these were further pursued jointly by Paley and Zygmund, who was resident at Cambridge in 1930–31. In the latter work Borel's *Calcul des probabilités dénombrables* was applied with surprising acumen to the construction both of existence proofs and of "Gegenbeispiele."

In order to continue these researches in collaboration with Wiener, Paley applied for an International Research Fellowship. Soon after his arrival in America, however, certain studies of lacunary series which Paley had already begun suggested a new attack on the theory of interpolation and allied trigonometrical problems. These results led successively to the study of quasi-analytic functions, of entire functions of order one-half, and of many related questions. The results of this work are forthcoming in the Transactions of the American Mathematical Society and will be incorporated in the Second New Haven Colloquium Lectures to be given by Wiener in 1934.

In view of the very short time which he had been on this continent, the impression which Paley had made on American mathematicians is remarkable in the extreme. He had won friends and scientific admirers everywhere. He was to have participated in the Symposium on Professor Fejér's lectures at Chicago this summer. We share with his English teachers and colleagues the feeling that his premature death is an irreparable loss to mathematics.

NORBERT WIENER

THREE BOOKS ON TOPOLOGY

Einfachste Grundbegriffe der Topologie. By Paul Alexandroff, with an introduction by David Hilbert. Berlin, Julius Springer, 1932. 48 pp.

Einführung in die kombinatorische Topologie. By Kurt Reidemeister. Braunschweig, Vieweg, 1932. xii+ 209 pp.

Knotentheorie (vol. 1 of the Ergebnisse der Mathematik und ihrer Grenzgebiete). By K. Reidemeister. Berlin, Julius Springer, 1932. vi+114 pp. and 114 figures.

To the uninitiate in matters of combinatorial analysis situs the first two of these books on topology will seem hardly to deal with the same subject, so completely do they complement each other. The little book of Alexandroff is an admirable and simple exposition of the ideas centering around homology and dimension, while Reidemeister's book, more complete and more important scientifically, is concerned with the study of abstract discrete groups and their topological applications.

Alexandroff has written particularly for those who do not care to undertake a systematic study of topology, and his enthusiasm in exhibiting the beauties of the subject will not escape the reader. There are many simple examples and the accompanying drawings are so skillfully made that one can not fail to see how the homology group operates, or to appreciate its intuitive meaning. The author does not allow the proofs of important matters to depend on the reader's intuition; on the contrary, it is remarkable how much he has treated in complete scientific detail. There is for example a very readable proof of the invariance of the Betti numbers (modeled after an elegant proof by Alexander) and a proof of the author's own important deformation theorem, with a corollary in the theory of dimensionality. The reader who wishes to learn more about these subjects will know where to begin from the numerous references to other authors; but for those who do not intend to consult the original papers, the references will not always give a complete nor accurate impression of the history of the subject. For example, the first formulation of the very important "Pflastersatz" was by Lebesgue in 1911, a fact that might well have been mentioned, despite the incompleteness of Lebesgue's proof. And again, what the author presumably means by the Lefschetz-Hopf fixed-point formula is surely due to Lefschetz, and should be named, we believe, accordingly.

For the purposes of his book, the author was undoubtedly wise in limiting himself to the homology theory; as for its importance, the author points out that it is this part of topology which will tend more and more to govern the development of the whole subject. This is undoubtedly true in the *unified* theory of general spaces, to which the author has made distinguished contributions. On the other hand there remains the fundamental problem of classifying manifolds, before which the homology apparatus, by itself, seems powerless; and even in the transformation theory which the author mentions, these methods seem to yield only broad variations of one special type of result. In these and other questions, one encounters not only the homology groups, which are

abelian and therefore relatively tractable, but also the much more unruly Poincaré groups which are defined by discrete sets of generators and finite sets of generating relations. One realizes the difficulties involved in the study of such groups from the fact that as yet there is no known general method of determining whether or not two symbols represent the same group element (the word-problem).

In recent years the works of Dehn, Nielsen, Reidemeister, Schreier, and others have brought about something like the beginnings of a general theory of discrete groups, and it is precisely to this theory that Reidemeister's *Einführung* is devoted. The first half of the book is purely group-theoretical. Here we find solutions of a number of special cases of the word-problem, and a number of results concerning the generation of subgroups, with Schreier's well known theorem, that any subgroup of a free group is free, coming out in various ways as a corollary. The remainder of the book deals with applications to one- and two-dimensional complexes and their covering complexes. This is the topological part, and the author has succeeded in making the treatment abstract throughout, in the sense that no limiting process is ever used or implied. An abstract reduction of surfaces to normal forms appears to be new and here the dual manifold turns out to be very useful. These parts are not easy to read. It seems to the reviewer that a few figures (there is not one) would have helped enormously, without compromising the abstractness of the treatment. It would have helped for example to see what the dual of a manifold which only contains one 2-cell really looks like; and in the definition of the representative graph (Gruppenbild) of a group, an example or two would have made it unnecessary for one reader, at least, to consult the original paper of Dehn as an introduction to the *Einführung*.

In his *Knotentheorie* Reidemeister is less severe with the reader, and various types of knot diagrams and operations on them are nicely illustrated. There are a number of ways of associating a matrix or a group with a knot, and the problem of classifying knots then becomes one of finding such calculable invariants of the groups and matrices as are also *knot invariants*. Practically all known results along these lines, including several hitherto unpublished, are included in this work, and the author's own contributions in which the interrelations of the various groups are developed serve effectively to unite the existing theory. The knot problem is perhaps the oldest problem in analysis situs. Gauss defined looping coefficients, and Listing, in the first published *Topologie* of mathematics, devoted much space to knots. Later, the knot problem assumed a temporary importance in physics. An atom was thought of as a knotted vortex in the ether and it was natural to suppose that the properties of an element depended on the knot structure of its atoms. More recently, it has been pointed out that the problem of classifying three-dimensional manifolds may be intimately connected with that of classifying knots. In one of Tait's classic papers on knots (1879), he admits that he "has not succeeded in catching the right note." Today we have a "theory" and have built much ingenious mathematics around the concept of knot. But what more do we actually know about knots themselves? The answer seems to be this: any knot whose plane projection contains nine or fewer double points can be recognized

as belonging to one of a finite number of non-equivalent types. This meager knowledge depends partly on the empirical results of Tait and his contemporaries, partly on the known calculable knot invariants such as those discovered by Alexander, and partly on special results (some of which are still unpublished) concerning pairs of knots which are not distinguishable by their invariants. Thus the problem is still open and still fascinating—the more so since it is now apparent that even though the problem seems to be one of abstract groups, progress may depend on the results of the most unexpected domains of algebra. The complete and concise little work of Reidemeister will do much to encourage further attacks.

P. A. SMITH

MOORE ON POINT SETS

Foundations of Point Set Theory. By R. L. Moore. American Mathematical Society Colloquium Publications, Volume 13. New York, 1932. viii +486 pp.

We are told in the Preface that this volume is intended to be a self-contained treatment of the foundations of the point-set-theoretic branch of analysis situs. It is concerned chiefly with those topics which are the results of Professor Moore's own research. Hence the book does not mention certain topics—dimension theory, for instance—which are closely connected with the topics discussed, but with the development of which Professor Moore was not primarily concerned.

The present treatment of point set theory is based upon a system of axioms, the undefined notions being *point* and *region*. General logical concepts are assumed, including in particular those of the logic of classes and the fundamental propositions concerning integers. Of these concepts, the author states explicitly only the Zermelo Axiom, which is to be considered as included among his assumptions. In addition to these introductory statements, the Introduction (pp. 1–4) is concerned with definitions of various types of sequences.

Chapter 1 (pp. 5–85) contains the theorems which can be derived from Axioms 0 and 1. The author also shows that most of these theorems follow from Axioms 0 and 1_0, where 1_0 is a weaker form of Axiom 1. Since Axioms 1 and 1_0 each consist of several parts, it is not surprising that so many theorems can be proved from them. While certain theorems can be proved with still weaker hypotheses, the author makes no attempt to do so. But the group of examples on pages 24–28 shows that in the case of certain theorems at least, no unnecessary restrictions have been placed in the hypotheses to compensate for the weakness of the assumptions concerning the space containing the sets. It is shown by these examples that under weaker hypothese the conclusions of this group of theorems become false, even if we assume that the underlying space is the euclidean plane.

The topics discussed in Chapter 1 are concerned with the following ideas, arranged roughly in order: boundary point, sequential limit point, Borel property, connectivity, irreducible continuum, limiting set, separation of two sets

by a third set, cut point, arc, link, indecomposable continuum, inner limiting set.

In Chapter 2 (pp. 86–151) are contained the consequences of Axioms 1 and 2. Some of the concepts defined and discussed in this chapter are: connectivity im kleinen, local connectivity, continuous curve, simple continuous curve, end point, acyclic continuous curve, regular curve, cyclic connectivity. From the standpoint of the study of continuous curves, Theorem 9 of this chapter is of importance, inasmuch as this theorem states that all the internal properties of a continuous curve may be derived from the fact that a continuous curve is a space satisfying Axioms 0, 1, 2.

Chapter 3 (pp. 152–174) discusses the consequences of Axioms 1–4. This chapter is principally concerned with simple closed curves and the sets complementary to them. The convention introduced on page 153 of selecting a definite point and calling it the point at infinity is a useful method to show that theorems concerning subsets of a sphere can be proved from corresponding theorems concerning subsets of a plane. If this convention is followed when the space S is a plane, the usual meaning of *exterior* and *interior* will be interchanged in the case of certain simple closed curves, and *bounded* and *unbounded* will not have their usual meaning. Hence proper care must be exercised in interpreting any theorems proved for the plane on the basis of this convention.

The theorems of Chapter 4 (pp. 175–323) are derived from Axioms 1–5. They may be divided roughly as follows: Theorems showing the existence of arcs bearing certain relations to given sets, theorems on separation by simple closed curves and by other continua, theorems on abutting and crossing of arcs, theorems on continuous curves and their complementary domains, theorems on accessibility, theorems on triods and related concepts, theorems on properties S and S' and their relation to uniform connectivity im kleinen. The chapter concludes with the proof of the Moore-Kline theorem on the conditions under which a closed set is a subset of an arc.

Chapter 5 (pp. 324–411) is entitled: *Upper semi-continuous collections.* The first part of this chapter is based upon Axioms 0, 1′, and C, where 1′ is a stronger form of Axiom 1. The second part is based on Axioms 1′, C, 2–6. Part 3 discusses equicontinuous collections of continua. Other concepts defined in this chapter are: graphatomic subset, order of a point, essential continuum of condensation.

In Chapter 6 (pp. 412–429) and Chapter 7 (pp. 430–461), the author shows that every space which satisfies a certain collection of the axioms is topologically equivalent to the number plane, and the substitution of Axiom 8′ for Axiom 8 gives a similar result for the number sphere. Theorems are also given on extending a homeomorphism between two sets to a homeomorphism between two sets containing the given sets. Finally the matter of introducing *distance* is taken up.

In an appendix (pp. 462–465) the author comments on certain theorems and their relation to the work of other mathematicians. The book concludes with an extensive bibliography and a glossary.

As in any book on the foundations of a branch of mathematics, the object of the author is to prove as much as possible from a minimum set of assump-

tions and to make his proofs as precise and as rigorous as possible. We shall now comment on a few minor inaccuracies which may serve to confuse the reader of the book.

The definition of a *sequence* on page 2, is phrased in a rather original manner. A sequence G is a collection of *sensed pairs*, whose elements are selected from a set M. (It might be noted parenthetically that a sensed pair, or ordered pair, is one of the most fundamental concepts underlying the use of logical symbolism.) The sensed pairs are the *elements* of the sequence G; the *terms* of the sequence are the elements of the set M. A term x is said to precede a term y in the sequence (although neither x nor y is an element of the sequence) if the sensed pair (x, y) is a member of the sequence. But in the final paragraph of page 4, when the author refers to the sequence $(A, 1)$, etc., the symbols are *not* intended to represent sensed pairs, as one might infer from the definition of sequence. The ambiguity inherent in this paragraph might have been avoided if the precise meaning of the symbols (A, i) and (B, j) had been given.

Sequence as thus defined means any type of arrangement in linear order. Various types of sequence are then defined: well-ordered sequence, infinite sequence, finite sequence, simple sequence. The reader is then told that *sequence* is to be used hereafter in the sense of simple sequence. It is unfortunate that some other word was not used for the general concept of sequence as first defined. It is rather confusing to use the same word for a general property and then for a very special case of that general property.

Example 6 on page 112 is incorrect, as the sets H and K described there have more than T in common. This example may be corrected as follows: For each n and i, $(1 \leqq i \leqq 2^{n-1})$, let B_{in} denote the point $((2i-1)/2^n, 1/n)$, and let C_{in} denote the point $((2i-1)/2^n, 0)$. Then with T as defined in the book, let H be T plus all the intervals $B_{in}C_{in}$ for i odd, and let K be T plus all such intervals for i even. The sets as thus constructed have the desired properties.

Theorem 90, page 271, is incorrect unless the fourth line be changed to read: "... mutually separated *connected* point sets ...".

Theorem 8, page 435, is partly false, as is shown by the author's own statement that a segment of an arc is topologically equivalent to an open curve. An open curve is a continuum, while a segment is connected but not closed. Hence the collection of all closed sets and the collection of all continua are not topological collections, and these statements should be deleted from the theorem.

Corrections to Theorems 6, page 434, and 19, page 451, are indicated below.

There are numerous misprints throughout the book, most of which are easily discovered and corrected by one making a careful study of the book. The following errors are worthy of noting here. On page 345, line 9, replace G by H. On page 352, line 2, replace the second D by U, and in line 3, replace k by n.

The usefulness of the book for reference purposes is marred by the fact that a number of definitions have been omitted from the glossary. Among them are: outer boundary (p. 193), Property A (p. 284), Axiom 6 (p. 368).

Certain definitions have been misplaced in the text. The definition of non-degenerate (p. 47) should precede the definition of totally disconnected (p. 30). The U^* notation is used many times before it is formally defined on page 110.

Usually its meaning is given, but such is not the cases on pages 91 and 92. Continuum of condensation is mentioned on page 76 but is not defined until page 106.

A different type of misplacement occurs in the case of the definitions of simple closed surface and cactoid. These are defined on page 151, but are not mentioned further until much later in the book.

Other definitions seem to have been omitted entirely from the book, such as simple domain and contracting sequence (p. 453).

There is a possibility of confusion on page 452 ff., because it is nowhere shown that a simple closed surface is a number sphere. In Theorems 20 and 21, S' is called a *space*; in the following theorems it is a *simple closed surface*, but the author has not shown that this change is justified.

But the reader of the book will probably be more concerned by the fact that the author does not discuss the significance of the axioms nor the logical interrelations between them. Questions such as the following will naturally occur to the reader, and we shall attempt to answer these questions below. (1) Why is Axiom 0 assumed in Chapter 1, but not thereafter? (2) There are two different propositions called Axiom 6. Are they logically equivalent? (3) The notation for Axioms 5, 5_1, 5_2 suggests some relation between these axioms. Just what is this relation? (4) Are there any further logical relations between the numbered axioms?

(1) The answer to this question is that the author has tacitly assumed Axiom 0 throughout the book and there is therefore no reason for including it in the title of Chapter 1 and omitting it from the titles of the following chapters.

For if we do not assume Axiom 0 to be true, we are confronted by the peculiar situation that a region is not necessarily a point set, but that all the definitions of Chapter 1 are worded on the assumption that a region *is* a point set. If in Chapter 2, we do not assume Axiom 0, our domain of discourse is the set of all points S and the set of all regions T. By Axiom 1, certain subsets of T *cover* S. But *cover* has been defined on page 5 for collections of point sets only.

Let us first suppose that it is permissible to define *cover* arbitrarily for collections which are not collections of point sets. It can then be shown that with the proper definition of *cover*, certain theorems of Chapter 2 are false. Hence not all the theorems of Chapter 2 follow from Axioms 1 and 2 alone, if *cover* be defined arbitrarily.

Let us then define *cover* thus: The collection G of sets is said to *cover* the point set M if each point of M belongs to some set of the collection G. With this definition, for any set X, let us denote by X' the point set consisting of all points which belong to the set X. Then if a point set M is covered by a collection G of sets (X), it is also covered by the collection G' of point sets (X'), corresponding to the sets (X) of G. Axiom 1 shows the existence of collections of regions for which not all of the sets (X') are vacuous. If we replace *region* R wherever it occurs by *point set* R' *consisting of all points contained in the region* R, all the statements of the book (including those of Chapter 1) are true whether Axiom 0 be assumed or not.

Rather than make such a replacement, it seems simpler to assume explicitly Axiom 0 throughout the book, and we shall do so hereafter. In that case

Axiom 0 should be included with the other axioms in the titles of Chapter 2 and the following chapters.

(2) It is easy to construct examples to show that Axiom 6 on page 368 is a weaker form of Axiom 6 on page 412 and that the theorem on page 434 is not true if the weaker form of Axiom 6 is used. Since the weaker form of Axiom 6 is used only in connection with the second part of Chapter 5, the easiest way to correct this matter is to denote the axiom on page 368 by 6_0 instead of 6. It might be noted again that Axiom 6_0 was overlooked in the construction of the glossary.

(3) Concerning Axioms 5, 5_1, 5_2, the following facts may be of interest. If Axioms 0, 1_0, 3 are true, then no region (and hence no domain) is degenerate. If Axioms 0, 1_0, 3, 5_1 are true, then Axiom 2 is true.

The title of Chapter 6 is *Consequences of Axioms* 1, 2, 4, 5_1, 5_2, 6, 7. However, in the proof of Theorem 2, a theorem is used whose proof seems to depend on Axiom 3. Thus the author seems to be assuming Axiom 3 also (and Axiom 0 of course), and in that case Axiom 2 may be omitted from the list of assumptions, as we have just shown. Hence it seems that the proper title for this chapter is *Consequences of Axioms* 0, 1, 3, 4, 5_1, 5_2, 6, 7.

From Theorems 2 and 3 of this chapter it follows that Axiom 5 is a logical consequence of Axioms 0, 1, 3, 4, 5_1, 5_2. This is a partial explanation of the notation used for these axioms.

(4) From the logical relations pointed out in the preceding section, we see that the hypothesis of Theorem 6, page 434, should read: ". . . Axioms 0, 1, 3, etc."

Since every subset of a compact set is compact, whenever Axiom 8′ is true, then Axiom 6 is true. Hence the hypothesis of Theorem 19, page 451, should read: ". . . Axioms 0, 1, 3, 4, 5_1, 5_2, 7, 8′ . . .".

In conclusion, the reviewer wishes to state that this book will undoubtedly prove to be an excellent text from which to obtain an insight into the nature of the problems considered by the school of mathematicians headed by Professor Moore. The book is a worthy addition to the set of volumes that have been published by this Society.

H. M. GEHMAN

SHORTER NOTICES

Die mathematische Denkweise. By Andreas Speiser. Zurich, Rascher, 1932. 137 pp.+9 pp. music.

This book is not a treatise on how mathematicians think. It is a collection of essays on mathematical thought as it is revealed in art and music, in philosophy and astrology. It is the work of a man of broad culture—one whose contributions to group theory are well enough known, but who is also at home in yet more esthetic realms and is conversant with the history of serious human thought.

The first chapter, *Delimitations*, while not attempting to define mathematics, sketches the history of its relations (in the few centuries preceding 1900, relations of decreasing intimacy) with philosophy and art, as well as with language, physics, and religion. There is good history, sound wisdom—a foretaste of a rewarding book. It is the symmetry and rhythm in art which is most closely akin to mathematics and especially to the mathematics of groups. (One might, of course, prefer to say that mathematics is artistic.) The love for such symmetry, for a neat formulation, sometimes leads Speiser to statements which are more suggestive than convincing. Thus, the triadic form of Proclus and Hegel is used to characterize center, radius, and circumference of a circle, and then: "Mathematics forms a triad with philosophy and art, wherein philosophy corresponds to the inert center, mathematics, as progressive research, to the πρόοδος, art as limiting and formative activity, which attains beauty by being driven back upon the center, to the ἐπιστροφή."

Two chapters are next devoted to *Symmetries in ornamental art* and *Questions of form in music.* In the former there are examples from many stages of history (unfortunately it was financially impossible to give the illustrations which would have facilitated the argument). In the latter the symmetries in simpler musical forms are discussed, and here we do find a number of themes reproduced, and a considerable portion of a Beethoven sonata analyzed. That symmetry does appear here and contributes greatly to the beauty of the work is undeniable. Yet, were that quality sufficient, we could look forward to a machine for the production of masterpieces. In fact, symmetry, in the strict mathematical sense, is intolerable. Speiser recognizes this in describing musical performance. "The nuances are by no means gratuities of sensitive souls, but the main pillars of the reproduction of a piece, since they alone reveal its symmetric content. The extraordinary accuracy which artist and listener bring into play is one of the clearest proofs of the mathematical structure of the human spirit"—but is not the need for these very nuances also a proof that mathematics is not enough? An amusing hypothesis is this—"perhaps the good work of art is characterized by a minimal property; it is the simplest piece consistent with the symmetry complex which it contains." There is another aspect of the affinity of mathematics with music, which Speiser might have mentioned. The satisfaction coming from these two arts is, in part, due to that temporal element which is lacking in visual art. A musical composition, a mathematical

proof, needs for its greatest esthetic effect a sense of suspense during its development, with the full resolution deferred until the end. If all good things come by threes, we might add the detective story as the third example of this type.

The later chapters deal with philosophy: Dante's philosophy of nature, Proclus Diadochus (the most admired of the ancient mathematicians) on mathematics, number and space with the Neo-Platonists, Goethe's color doctrine, astrology. In all these essays we have an author who has the advantage (the word of course reveals prejudice) of the newer empirical point of view, and yet gives most sympathetic accounts of those idealists who believed that cogitation would disclose the truth about reality. We have, for example, Dante's explanation of why all inhabited land should be in the form of a lunette—with its center at Jerusalem—exactly opposite the divine point of heaven. Originally the Godhead attracted the bulk of solid matter to the opposite side of the sphere. But "Lucifer plunged headlong down from this divine point. When the earth saw him coming, it shrank back aghast. Lucifer was held fast in the center, and above his three heads there appeared a cave, Hell; then earth closed behind him again, and thus there arose the Mountain of Purgatory near the Antipodes." If it is a little less than fair to cite only this example of the working of reason uncontrolled by experiment, we would atone for it by urging that the book itself be read. It is not a necessity for one's library; it is a delight.

E. S. ALLEN

Lezioni sulla Teoria delle Superficie Algebriche. By Federigo Enriques. Raccolte di Luigi Campedelli. Parte 1a. Padova, Antonio Milani, 1932. 4+481 pp.

The book contains 481 pages and comprises the first part of a volume assembled by Dr. Luigi Campedelli in collaboration with the author. It is not made clear, however, whether the work is presented as a preliminary edition of a text or as a set of organized lecture notes.

A general introduction of twenty pages embraces birational transformations, examples of singularities, geometry upon surfaces, and exceptional curves. Among this material occurs the fundamental theorem to the effect that a surface F, having any singularities whatever, may always be transformed, by means of a birational transformation, into another surface having no singularities.

Without listing the actual contents it may be said that there are five chapters embracing sixty-six paragraphs. The chapter topics are: systems of linear curves, systems of covariants and invariants, adjoint surfaces, the genus number and the theorem of Riemann-Roch, classification of surfaces, in particular, regular surfaces.

Chapter 1 contains illustrative examples of the topics under discussion; the same technique is observed in Chapter 3; Chapters 2, 4, 5 have appended bibliographies. Chapter 5 embraces more than one third of the book and includes an extended discussion of canonical surfaces and curves, Cremona transformations, rationality of surfaces and planes in connection with the theorems of Noether and Castelnuovo.

The book has apparently been photostated; the title, chapter headings, etc., are hand printed while the body of the text is in excellently executed script.

H. R. PHALEN

Bericht über neuere Untersuchungen und Probleme aus der Theorie der algebraischen Zahlkörper. By H. Hasse. Teil I. Klassenkörper. Teil I^a. Beweise zu Teil I. Teil II. Reziprozitätsgesetze. Leipzig, Teubner, 1930. 134+4+204 pp.

In a series of articles in the Jahresbericht der Deutschen Mathematiker-Vereinigung, Hasse has given a general report on the recent development of the theory of class-fields and the laws of reciprocity. These articles have now appeared in a separate edition and form a valuable addition to Hilbert's report on the theory of algebraic numbers. It would be highly desirable to have similar modernizations of other parts of Hilbert's report.

The first part contains mainly a simplified representation due to Hasse of Takagi's and Furtwängler's principal results, proving the existence of class-fields corresponding to any class division of the ideals in an algebraic field, and the completion theorem, that every relative abelian field can be considered as a class-field for some class-division in the fundamental field. There are also applications of the general theory to the theorem of Kronecker-Weber, that every absolute abelian field is a subfield of a cyclotomic field, and also to the "Jugendtraum" of Kronecker, that is, the representation of all abelian fields relative to a quadratic imaginary field by means of singular moduli.

In Part 2, Hasse proves the general law of reciprocity of Artin and applies it to prove the various special laws of reciprocity. One also finds various applications to ordinary theory of numbers, particularly to Fermat's theorem.

It is impossible to give a detailed discussion of the report, without using a great number of technicalities. It may suffice to say that it is a remarkable piece of work and that it will be a sourcebook for the future work in this field.

OYSTEIN ORE

Einführung in die Theorie der Wellengleichung. By Harry Schmidt. Leipzig, J. A. Barth, 1931. vi+146 pp.

This book gives in a short compass and in painstaking manner an account of those topics in differential equation theory which are now commonly regarded as essential for graduate students of theoretical physics. The book is a useful introduction to the more ample, world-renowned books by Courant-Hilbert and Frank-von Mises. In ordinary differential equation theory a short and good account of Fourier series and Fourier integrals is given and a few pages are given to the Sturm-Liouville boundary-value problem. For two-dimensional problems the usual method of reduction to one dimension by separation of the variables is followed and the gamma function and Bessel functions are discussed (in addition to Fourier series in two independent variables). The Fuchs theory of singular points of ordinary linear differential equations is treated and Legendre polynomials are discussed as illustrating this theory. A final chapter contains a discussion of the Schrödinger wave equation.

The book will evidently be very useful. One could criticise it as too pedantic and object to the heavy pages of formulas. The systematic avoidance of vector analysis and of geometric references helps to produce this impression. No reference is made to Riemann's method of integration for hyperbolic equations. The printing maintains the high standard set by the publisher.

F. D. MURNAGHAN

Constitution of Atomic Nuclei and Radioactivity. By G. Gamow. Oxford, Clarendon Press, 1931. viii+114 pp.

This book is one of the International Series of Monographs on Physics and gives an account of the knowledge of atomic nuclei at the time it was written (May 1931). Except in Chapter IV, where an interesting account of the theory of inelastic collisions, according to wave mechanics, is given, very little mathematics is used. The following chapter headings give an idea of the contents: The constituent parts and energy of nuclei. Spontaneous disintegration of nuclei. Excited states and electro-magnetic radiation of nuclei. Artificial transformation of nuclei.

F. D. MURNAGHAN

Vorlesungen ueber Funktionentheorie. Part 2. By Alfred Pringsheim. Leipzig, Teubner, 1932. xiii+498 pp.

The volume under review is the last part of the author's *Vorlesungen ueber Funktionentheorie,* and the final section of the two volume *Vorlesungen ueber Zahlen- und Funktionenlehre,** and is devoted to a discussion of the properties of single-valued analytic functions. Coming as it does as the second section of a work on infinite series, it is quite natural that this treatise on analytic functions should be dominated by the Weierstrass angle of approach, namely, that of power series. In fact the author set himself the goal of presenting this theory exclusively from this angle, so far as possible. As a consequence the choice of material presented is dictated by the method employed. However, one cannot help but be impressed with the results obtainable, and the power of the method.

To give a brief summary of the contents, one finds in the first chapter a treatment of infinite products of functions, especially of the linear variety, the Weierstrass product theorem as applied to entire functions, and its application to the factorial and gamma functions, winding up with the extension of the Weierstrass product theorem to the case in which the zeros of the function are isolated, but may have continuous arcs as derived set.

The second chapter gives a systematic treatment of entire functions of a finite order, and derives beautiful results giving the relationship between the order of the function, the behavior of the coefficients of the power series, and the convergence properties of the reciprocals of the zeros of the function. It is gratifying to find these interrelations collected in one place, and so clearly presented. The chapter also contains a proof of the Picard theorem for entire functions of finite order, based on the result that any change in a finite number of coefficients of an entire function of finite order with a finite number of zeros makes it an entire function with an infinite number of zeros, and emphasizing the unusualness of the existence of the exceptional values mentioned in the theorem.

The third chapter treats of fractional transcendental functions, giving the Mittag-Leffler theorem for the case of simple poles, and leading up to an extensive treatment of the properties of doubly periodic functions. The last part of the chapter discusses the conditions that a function have on its circle of

* Reviewed in this Bulletin: Part I, 1: vol. 25 (1919), p. 470; part I, 2 and 3: vol. 28 (1923), pp. 63–65; part II, 1: vol. 32 (1926), pp. 551–554.

convergence a single pole of the first order and applies the results to the Heine series. The chapter concludes with the use of continued fractions of the form

$$\frac{a_0}{1+}\ \frac{a_1 z}{1+}\ \frac{a_2 z}{1+}$$

for the representation of analytic functions.

The fourth chapter is devoted to single-valued functions with arbitrary singularities, giving the Mittag-Leffler theorem, and extending it to the case in which the singular parts are known at an isolated set of points, whose derived set may be continuous arcs. Special consideration is given to functions expressed in the forms

$$\sum c_n\left(\frac{1}{x-a_n}-\frac{1}{x-a_n'}\right) \quad \text{and} \quad \sum\frac{c_n}{x-a_n}.$$

The last part of the chapter is concerned with the conditions that certain points on the circle of convergence be singular points of a certain type, indicates the nature of series for which the circle of convergence is a singular line, and also gives a discussion of the Taylor expansion for one real variable.

The final chapter is concerned with complex integration. Up to this point the author has limited himself to a kind of integration, a mean value, defined by the values of a function on a circle of radius r, namely,

$$\lim_n \frac{1}{2^n}\sum_{m=1}^{2^n} f(c_n^m r)$$

where c_n is the fundamental root of $x^{2^n}=1$. The second chapter uses an extension of this mean value, which is something of the nature of an integral over an arc of a circle. But complex integration as such has been avoided and barred. So this last chapter is concerned with this subject, and some of its uses. But even here, power series are dominant, and used in the proof of the Cauchy theorem. The author applies complex integration in three projects, the uniform approximation of single-valued analytic functions by polynomials in star shaped regions, the relation of the singularities of $\sum a_n b_n x^n$ to those of $\sum a_n x^n$ and $\sum b_n x^n$, and the discussion of the function represented in the form

$$\int_0^\infty \tau^{-1/2}(\tau+1)^{-1/2}(\tau+x)^{-1/2}d\tau,$$

leading up to the proof of the Picard theorem for any transcendental entire function. The closing paragraph gives another proof of the Cauchy-Goursat theorem, and indicates that it may be made the basis of the theory of analytic functions.

The book is a model of clear, accurate, and careful exposition. Matters are taken up in logical order, and there is a definite appreciation of the possibilities of building up to climaxes. In his desire to be absolutely clear, the writer sometimes sins on the side of too much detail. But the result is a volume which can be read and understood by any one knowing the elements of the theory of analytic functions, and which gives a wealth of information on many important and interesting parts of the theory.

T. H. HILDEBRANDT

Contributions à l'Étude du Coefficient de Déformation des Fonctions Non Sinusoïdales. By Ernest Abason. Bucarest, Institut National Roumain, 1927. 125 pp.

General formulas are derived for the coefficients in the Fourier series expansion of a function $f(x)$ formed by joining a finite number of arcs, each of which may be represented by a polynomial in x. The results are expressed in terms of the discontinuous changes or "sauts" in $f(x), f'(x), f''(x), \cdots$, which occur at the abscissas at which adjacent arcs are joined. The calculations require the summation of finite trigonometric series, and this may be performed either numerically or graphically. Illustrative applications are given where $f(x)$ is a function formed by joining straight lines and also where the function $f(x)$ is used as a device to approximate a wave such as might be obtained by an oscillographic record of a physical phenomenon. It is pointed out (e.g., page 106) that in applications of the former type, the method is less complicated than the classical method of computing Fourier series coefficients, which, of course, is to be expected since the classical method requires in effect for the special cases considered a repetition of the integrations performed by the author in deriving his method. This work should be of considerable interest to those concerned with empirical harmonic analyses of waves.

W. R. BENNETT

The Size of the Universe. By L. Silberstein. Oxford University Press, 1930. viii+215 pp.

The aim of this book is to show that Einstein's assumption as to the metric of space-time is not valid, and to indicate what is the curvature of the alternative de Sitter metric in which space-time is supposed to have constant negative curvature. The first part (52 pp.) of the book gives a brief account of tensor analysis and elementary projective geometry. The second part (32 pp.) deduces Einstein's and de Sitter's solutions. Part III (17 pp.) is a somewhat bitter criticism of Hubble's estimate of the radius of space. Part IV (25 pp.) treats the geodesics and light rays of a de Sitter space-time. Part V discusses the Doppler effect and attempts to arrive at an estimate of the radius of curvature. The procedure adopted is not very convincing. A certain formula is arrived at, giving the Doppler effect of a star and involving two undeterminable constants, namely, the distance r of the star, and its perihelion distance r_0. Since nothing is known about r and r_0 a lot of observations are made, and it is assumed that all values of the ratio r_0/r in the interval 0 to 1 are equally probable. This logic is reminiscent of the argument that since we do not know at all whether Mars is inhabited or not, the probability that it is inhabited is $\frac{1}{2}$. Anyhow, the author arrives at the estimate $6.27 \cdot 10^6$ light years for the radius of elliptic space (from an examination of the data from 459 stars). The book is well printed and bound, and makes stimulating reading, but a conservative reader may be pardoned for thinking, on laying it down, that as regards the size of the universe, one man's guess is as good as another's. Anyone interested in the topics discussed in this book would be well advised to read the authoritative and readable report by Robertson, which appeared in the January, 1933, issue of Reviews of Modern Physics. It is stated in this report that Silberstein's estimates are generally regarded as impossibly low.

F. D. MURNAGHAN

L'Idée Générale de la Mécanique Ondulatoire et de ses Premières Applications. By Marcel Boll. Paris, Hermann, 1932. 74 pp.

Théorie de la Quantification dans la Nouvelle Mécanique. By Louis de Broglie. Paris, Hermann, 1932. viii+250 pp.

The first of these books attempts to give in very short compass and for non-specialists an idea of the general principles underlying wave mechanics. On the whole the attempt is successful, and the very brevity and lack of detail involved have the merit that a reader will wish to study more detailed treatments after having read this little book. The following chapter headings indicate sufficiently the topics treated. I. La liason entre la mécanique et l'optique. II. Les relations d'incertitude. III. Les niveaux d'énergie. IV. Valence et affinité chimiques. V. La conduction électrique des métaux.

The book by de Broglie is an authoritative account of the subject by one of its creators. After an historical introduction, the first chapter gives a clear presentation of the connection between mechanics and geometrical optics, and ends with the general wave equation for a single particle. The second chapter gives the probability interpretation of the wave function and Heisenberg's uncertainty relations. Chapter III shows how to treat a dynamical system made up of a number of particles, and Chapter IV describes Schrödinger's method and Heisenberg's matrices. The rotator and simple harmonic vibrator (in one, two, and three dimensions) are treated in Chapter V, and Chapter VI deals with the hydrogen atom. Chapter VII, which concludes the first part of the book, discusses briefly the method of perturbations and the principle of interference.

The second part of the book is devoted to more abstract methods. Chapters VIII and IX treat, respectively, of real and complex vectors in a space of n dimensions, and introduce the matrices, orthogonal and unitary, which correspond to a transformation of coordinates. Chapter X extends these notions to functional space in which a vector, or point, is an integrable complex function of one or more real variables. The concept of a system of orthogonal functions is introduced, but no real mathematical treatment is given. It is simply assumed that all the functions discussed are developable in terms of the orthogonal functions considered. The same lack of mathematical detail shows itself in the discussion of the infinite matrices which correspond to a change from one set of orthogonal functions to another. Thus (p. 131) in introducing the product of two infinite matrices, an interchange in the order of summation, as if the matrices were finite, is made without comment. Chapter XI discusses matrices whose characteristic numbers include all the points of an interval and introduces Dirac's delta function. The reviewer feels that it is high time that the elements of Stieltjes integration be taught to physics students, so that such paradoxes as those concerned with Dirac's function may be avoided. The last chapters treat systems with constant external fields, mean values, the uncertainty principle, operators independent of the time, non-conservative systems, operators dependent on the time, and the general theory of first integrals.

On the whole, this is perhaps the best introduction to wave mechanics which has yet appeared. It is much less difficult than Dirac's book, and after reading it, Weyl's *Gruppentheorie und Quantenmechanik* should prove much easier to study. The appearance of the volume is all that could be desired.

F. D. MURNAGHAN

Cours de Mécanique Rationnelle. By J. Chazy. Tome I: Dynamique du Point Matérial. Paris, Gauthier-Villars, 1933. 392 pp.

This volume presents the first part of the course given by Professor Chazy of the Faculté des Sciences de Paris since 1928. Except for the subject of kinematics, it covers essentially the requirement for the Certificat de Mécanique Rationnelle.

The first chapter is devoted to the theory of vectors and develops the necessary algebra and calculus of vectors. Throughout the rest of the book vector methods are used exclusively.

The course is a logical development of newtonian mechanics, although the name of Newton is not mentioned in connection with the fundamental assumptions. To begin with, two systems of axes are defined as follows. The system of axes of Copernicus is a system of axes of coordinates of which the origin is the center of gravity of the solar system and the directions of which are fixed with respect to the fixed stars. A system of axes of Galileo is any system of axes having uniform rectilinear translation with respect to the axes of Copernicus. It follows that the acceleration of a moving point is the same with respect to either the axes of Copernicus or the axes of Galileo.

After making these definitions, the author then states four principles:

I. The principle of inertia or the principle of Kepler. An isolated material particle has uniform rectilinear motion, or zero acceleration.

II. The principle of Galileo or of initial conditions. At a given instant the points of a material system may be placed in given positions with given velocities; the later motions of the system, and in particular the initial accelerations, are completely determined.

III. The principle of the equality of action and reaction.

IV. The principle of the composition of forces (parallelogram law).

It is the second of these principles that appears to be unusual. It amounts to saying that the rectilinear motion of a single point is determined by a differential equation of the second order of the form $x''=f(x, x', t)$, together with arbitrary initial values of x and x'. In justification of this statement as a fundamental assumption, the author points out that there was a period in history when it was maintained, in effect, that the equation should be of the form $x'=f(x, t)$ and, on the other hand, it is conceivable a priori that the equation might have the form $x'''=f(x, x', x'', t)$.

If one were disposed to insist on satisfactory statements of fundamental principles before proceeding further, he might never get beyond the second chapter. But, having accepted the dynamical equations for the motion of a material particle, he will find in the remainder of the book a clear presentation of the usual topics. Particularly good is the discussion of various types of motion from the qualitative point of view.

It would appear to be impossible to lecture on theoretical mechanics without solving some specific problems by way of illustration. The author has, however, cut the number to a minimum and there are no problems in the text to be solved by the student. An appendix contains the questions given in the examinations for the Certificat de Mécanique Rationnelle de la Faculté des Sciences de Paris from June, 1928, to June, 1932, inclusive.

W. R. Longley

La Méthode de Corrélation. By R. de Montessus de Ballore. Paris, Gauthier-Villars, 1932. 77 pp.

This book presents a compact elementary treatment of the theory of correlation. The development presents very little that is novel. It is suggested that r^2, where r is the correlation coefficient, be used as a measure of correlation. The reason given for this change is that there is a tendency to assign too much importance to a small correlation, say to $r=.3$. If we used $r^2=.09$ for the same data, it is held that less significance would be attached to the correlation. But it seems to the reviewer that both r and r^2 should be given significance in relation to their probable errors and then surely r^2 is not easier to interpret than r. Illustrations of the method are given by applications to business statistics and to meteorological data. The reviewer has long felt that certain problems of meteorology should be more generally attacked by recent methods of statistical analysis.

H. L. RIETZ

Die Arithmetik in strenger Begründung. By Otto Hölder. 2d edition. Berlin, Springer, 1929. 73 pp.

This little book is a second edition of the author's Leipzig Programmabhandlung, published in 1914. It presents a genetic treatment of the real number-system and its arithmetic which, after the natural numbers have been set up, is clear, concise, and rigorous. The Dedekind cut definition of the irrational is used. There is a section on magnitudes as applied to the measurement of line-segments.

Since there has been considerable discussion of the foundations of arithmetic since 1914, one naturally looks to see what influence this discussion has had on the second edition. The only changes in the text are simplifications of proofs not related to essential problems of the foundations. It seems to this reviewer that a treatment of the number-system which does not consider explicitly the notions of order, mathematical induction, and finiteness cannot lay claim to being a rigorous foundation. Since this book chooses to leave these questions to one side, it hardly justifies the author's slighting reference in the preface to the work of the intuitionists.

D. A. FLANDERS

Höhere Mathematik. By Rudolf Rothe. Volume 4: *Uebungsaufgaben, Formelsammlung.* Heft 1. Leipzig und Berlin, B. G. Teubner, 1932.

The exercises in this pamphlet parallel the first part of Volume 1 of this series, to which it forms a supplement. They cover topics relating to variables and functions, and the fundamental operations of the differential and integral calculus, including maxima and minima and Taylor's formula. The examples are varied and many of them are suggestive, especially the ones dealing with functions and limits. The pamphlet can be recommended to the teacher of calculus who is looking for problems different from the usual variety, as well as those which can be given as thought provoking to the more intelligent student.

T. H. HILDEBRANDT

Mathematische Grundlagen der Quantenmechanik. By Johann von Neumann. (Die Grundlehren der Mathematischen Wissenschaften, Band 38.) Berlin, Springer, 1932. 262 pp.

The contents of this book are well described by its title if this is taken to imply, not an exposition of the mathematical technique of working quantum mechanical problems, but a thorough and logical development and discussion of the mathematical axioms on which quantum mechanics is founded. Its purpose is essentially the same as that of Dirac's well known book; its procedure, however, is markedly different. While Dirac presents his reasoning with admirable simplicity and allows himself to be guided at every step by physical intuition—refusing at several places to be burdened by the impediment of mathematical rigor—von Neumann goes at his problem equipped with the nicest of modern mathematical tools and analyzes it to the satisfaction of those whose demands for logical completeness are most exacting. A treatment of this sort is likely to be tiresome for the reader, but it must be said with emphasis that von Neumann has excellently avoided this danger, for his style is luminous and vivid, and has sufficient intrinsic momentum to carry the reader through the more intricate points of the analysis.

The proof of the equivalence of the theories of Heisenberg and Schrödinger is conducted in a manner more fundamental than is customary in physical textbooks on the same subjects. It is carried over into Hilbert space where the elements of the two theories become alike, and where the entire structure of the so-called transformation theory takes a pretty and simple form. As the author points out, this method circumvents numerous analytical flaws of the better known elementary theories and avoids, in particular, the use of the self-contradictory δ-function introduced by Dirac. To acquaint the reader with the properties of abstract Hilbert space, a fairly lengthy chapter on this subject is inserted.

Physical applications are usually omitted, except for two instances which are treated with interesting and illuminating detail: Heisenberg's principle of related indeterminacies (vulgarly denoted in English by the misleading term "uncertainty principle", which was apparently invented to baffle philosophers) and Dirac's theory of radiation.

There are two important respects in which this book is far superior to the treatments of quantum mechanics which have come to the reviewer's notice. The first is a serious endeavor to expose the status of causality. Von Neumann takes the view that causality consists in the possibility of inferring the results of future measurements from the results of past observations. On this basis he shows that the statistical implications of quantum mechanical axioms refute causality. In this connection proper attention is drawn to the twofold possibility of changes in the state of physical systems: first the "regular" continuous change of ψ-functions governed by Schrödinger's time equation; second the abrupt statistical change caused by measurements. The latter is discussed with interesting earnestness—not glossed over because of its difficulties of comprehension—and finally reduced, it appears, to the insolvable mystery of the correlation between physical stimulus and sensory consciousness.

The second point of unusual merit in von Neumann's treatment is that it

goes to the trouble of discussing quantum mechanical axioms in connection with statistical assemblies of systems. Clearly, if theorems like that of related indeterminacies are to have meaning in terms of measurements at all, they must be applied to large collections of similar systems. The material presented in relation to this matter is of course largely that published in the author's earlier papers and includes his treatment of thermodynamics. On the whole, the book is probably the most complete and rigorous discussion of the subject indicated by its title.

Naturally, no book is free from minor defects, and no review is complete without their exposition. We feel that von Neumann's book could have been improved by adding an alphabetical index. It also contains no foot-notes. All notes are collected in the appendix, and it happens occasionally that the reader, already disconcerted by having to turn many pages in order to find a reference given in the text, is referred again to a previous note to secure an insignificant bit of information.

HENRY MARGENAU

Wave Mechanics. Elementary Theory. By J. Frenkel. Oxford, Clarendon Press, 1932. viii+278 pp. $5.00.

This is one of the international series of monographs on physics edited by Fowler and Kapitza. Planned originally as a translation of the author's relatively brief *Einführung in die Wellenmechanik*, the text has been so expanded in revision that the completed work is expected to consist of three volumes, of which this is the first. The second is already in press, and the third is planned for next year.

The present volume is intended to give a general survey of the whole subject, using only elementary mathematics. The approach is from the physical rather than from the mathematical point of view, and the author succeeds in covering a vast amount of material. Starting with a brief historical survey of the corpuscular and wave theories of light and a glance at the relativity theory, he discusses the extension of the dual particle-and-wave concept to matter, and leads up to the wave mechanics through Heisenberg's uncertainty relation. A chapter on the wave mechanics of the motion of a particle in a field of force, in which considerable emphasis is laid on the properties of potential barriers, is followed by one on the wave mechanics of a system of particles. Next comes a chapter on statistical mechanics, in which the Einstein-Bose and Fermi-Dirac statistics are contrasted with the classical theory, and the book closes with a chapter on the application of the quantum statistics to the electron theory of metals, heat motion, and radiation. In this last chapter, the author adds to the familiar electron, proton, and photon a new particle, the phonon.

The author's comments and discussion, rendered in a most vivid style, are very illuminating and form the most valuable feature of the book. On the other hand, the book lacks the beautifully logical development of the subject exhibited in de Broglie's *Théorie de la Quantification*, and the reader, unless he already possesses an expert knowledge of the subject, is likely to emerge a trifle bewildered by the complexity and range of the material covered.

LEIGH PAGE

Elementary Mathematics from an Advanced Standpoint. Arithmetic, Algebra, Analysis. By Felix Klein. Translated from the third German edition by E. R. Hedrick and C. A. Noble. New York, Macmillan, 1932. 10+274 pp.

Klein possessed in an unusual degree the abilities of a great mathematician and the gifts of an inspiring teacher and lecturer. He had a broad knowledge of mathematics and a correspondingly deep insight into the foundations and interrelations of its various branches. Both Klein's qualifications for writing a book of this nature and the scarcity of such books combine in directing attention to the present volume.

This book, a translation of the first of Klein's three volumes entitled *Elementarmathematik vom höheren Standpunkte aus*, is a series of lectures that Klein gave for teachers of mathematics in secondary schools. The material is presented under the headings of arithmetic, algebra, analysis, and a supplement. The section on arithmetic treats the extensions of the number system and the laws of operation, beginning with integers and ending with complex numbers and quaternions. The treatment seeks to explain the *how* and *why* of the subject. As an example, we note the discussion of the little understood rule of signs: "minus times minus gives plus." The section on algebra is devoted to the solution of equations. First, some geometric methods are explained for investigating the real roots of rational integral equations containing parameters. Then complex roots are considered, especially of those equations whose solutions lead to a consideration of the groups of motions connected with the regular bodies. Free use is made of Riemann surfaces and other parts of the theory of functions of a complex variable. The section on analysis is devoted to the logarithmic, exponential, and trigonometric functions, and a discussion of the infinitesimal calculus proper. A wide variety of subjects is treated, however, in connection with these general topics: the construction of the early logarithmic and trigonometric tables, expansions in Fourier series, Taylor's Theorem, and Newton's and Lagrange's interpolation formulas will serve as samples. Finally, the supplement contains proofs of the transcendence of e and π, and a discussion of assemblages.

The real excellence of the book, however, is due to certain clearly defined characteristics of the presentation. In the first place, the historical development of the theory is traced. This is not history for history's sake alone, but history as an aid to gaining a deeper insight into the present state of the theory. In this connection it should be stated that the inductive method of presentation is used exclusively.

Secondly, the geometric aspects of the subjects treated are emphasized. It is significant that the book contains 125 figures. The geometric meaning of Fermat's Theorem is explained; the Pythagorean number triples are obtained by a geometric method. The graphs of the approximating polynomials of Taylor's series expansions are drawn in order to show the nature of the convergence and divergence; similarly for Fourier series. Klein would develop geometric intuition and sense perception as an aid to mathematical investigation.

Again, Klein shows the mutual relations between problems in different fields. His ability to discover such relations is well known, and many examples are to be found in this volume. We may mention the parallel treatment of the

logarithmic, trigonometric, and hyperbolic functions, one which incidentally robs Euler's formula, $e^{i\theta} = \cos\theta + i \sin\theta$, of its mystery.

Finally, the book is in the form of lectures. Somewhat informal, it is all the more vigorous and interesting. We hear Klein as he addresses his class, explains a drawing or a model, or hands them an early book for examination; we feel the inspiration that comes from listening to a great teacher.

The paper, printing, and binding are of high quality. The reader will notice a number of minor typographical errors, but they are largely misspellings of words that do not cause any inconvenience.

This book is familiar to many already in the original, and this review is written in the hope of introducing it to a still wider circle of readers among our students and teachers of high school and college mathematics. Our thanks are due to Professors Hedrick and Noble, who have rendered a service to mathematics by making this valuable book accessible to all who speak English.

G. B. PRICE

L'Hydrodynamique et la Théorie Cinétique des Gaz. By Y. Rocard. Paris, Gauthier-Villars, 1932. 160 pp.

The fluid of hydrodynamics is a continuum, while the kinetic theory views a fluid as formed of discrete molecules. Maxwell, Boltzmann, and Lorentz laid the foundation for bringing these two different methods of approach into harmony, and, more recently, their work has been carried on by Brillouin, Chapman, Enskog, and others. The two principal problems are to obtain the equations of hydrodynamics for real fluids from the kinetic theory, and to calculate the coefficient of viscosity by considering the molecular encounters.

The author has made important contributions to this subject himself, and has performed a very useful service in collecting into a single volume the results of all these investigations. This book is based upon lectures by the author in 1929 and upon a memoir published in the Annales de Physique in 1927.

E. P. ADAMS

Caractéristiques des Systèmes Différentiels et Propagation des Ondes. By Tullio Levi-Civita. Leçons rédigées par G. Lampariello. Traduction de l'italien par M. Brelot. Paris, Librairie Félix Alcan, 1932. x+114 pp.

In 1930 Professor Levi-Civita published two papers on the characteristics and bicharacteristics of Einstein's gravitational equations, and it was probably during the preparation of these papers that he realized the need of a clear and concise account of the theory of characteristics such as is given in his little book in the Italian language which was reviewed in this Bulletin of May, 1932.

Judging from the papers of Racah and Lampariello, Levi-Civita's work has been very stimulating to Italian mathematicians and so it is good news to find that his lucid and timely exposition has been made available to a greater number of readers by M. Brelot's excellent translation.

The use of large print for names of authors, heavy type for vectors, and a familar notation, all make the book very readable.

H. BATEMAN

NOTES

A new journal, Scripta Mathematica, devoted to the history and philosophy of mathematics, has been established under the auspices of Yeshiva College, New York City; Professor Jekuthiel Ginsburg, of that College, is editor-in-chief. The editorial board includes also R. C. Archibald, L. C. Karpinski, C. J. Keyser, Gino Loria, Vera Sanford, Lao G. Simons, and D. E. Smith.

The Annali della Reale Scuola Normale Superiore di Pisa announces the following subjects for a prize memoir, to be submitted before June 15, 1934: a brief summary of the theory of partial differential equations of elliptic and parabolic type; a brief summary of the theory of analytic functions of several complex variables and of important special classes of such functions. Competing memoirs should be unpublished, written in Italian, French, English, or German, and not longer than the equivalent of thirty quarto printed pages. The prize memoir will be printed in Italian in the Annali.

The Lazzaro Fubini prize for 1932 has been awarded to Dr. Tullio Viola, of the University of Bologna, for several memoirs and notes, including a memoir *Sui diagrammi reciproci del Cremona* and several papers on unilateral continuity and derivatives. An additional prize was awarded from the same foundation to Dr. Mario Villa, of the University of Pavia, for his work in algebraic geometry.

The Henry Draper medal of the National Academy of Sciences has been awarded to Dr. V. M. Slipher, astronomer of the Lowell Observatory, Flagstaff, Arizona.

Professor U. Amaldi, of the University of Rome, has been elected a corresponding member of the Pontifical Academy of Sciences.

Professor Albert Einstein has received an honorary doctorate from the University of Glasgow.

Professors B. Finzi, of the University of Milan, and R. Serini, of the University of Pavia, have been elected corresponding members of the Reale Istituto Lombardo.

Professor Jules Haag, of Besançon, has been elected correspondent of the Paris Academy of Sciences in the section of mechanics, as successor to Professor H. Villat, of the Sorbonne, elected a member of the Academy in the same section.

The University of Paris has conferred an honorary doctorate on Professor Tullio Levi-Civita, of the University of Rome.

Professor J. A. Schouten of Delft has been elected a member of the Royal Society of Amsterdam (Koninklijke Akcademie van Wetenschappen, Amsterdam).

Professor A. Signorini, of the University of Naples, has been elected a resident member of the Royal Society of Naples, and Professor M. Pascal a corresponding member.

At the April meeting of the National Academy of Sciences in Washington, Professors G. C. Evans, of Rice Institute, and J. F. Ritt, of Columbia University, were elected members of the Academy. At this meeting the following mathematical papers were read: *Geometry of the Laplace equation*, by Professor Edward Kasner; *The geometry of spinors*, by Professor Oswald Veblen.

Professors Jesse Douglas and M. H. Stone have been elected fellows of the American Academy of Arts and Sciences.

Professor E. V. Huntington, of Harvard University, has been elected a member of the American Philosophical Society, and Professor Max Planck, of Berlin, a foreign non-resident member.

Professor W. H. Roever, of Washington University, has received a grant from the Rockefeller Science Research Fund for the publication of some of his work in descriptive geometry. Part of this work was done for the Committee on Standards for Graphical Presentation, on which Professor Roever served as representative of the American Mathematical Society.

Dr. M. F. Deuring and Mr. Saunders MacLane have been awarded Sterling fellowships in mathematics at Yale.

Professor Enea Bortolotti, of the University of Cagliari, has been promoted to a professorship of analytical geometry.

Dr. Fritz Lettenmeyer has been promoted to an associate professorship of mathematics at the University of Munich.

Professor Theodor Vahlen, of the Vienna Technical School, has been appointed professor of mathematics at the University of Greifswald.

On the occasion of the visit of Professor Neils Bohr to Pasadena in May, the department of physics of the California Institute of Technology conducted a symposium on the general subject of the physics of nuciei and high energy transformations. Professor Bohr delivered a series of lectures on the foundations of atomic mechanics.

Dr. H. F. Bohnenblust has been appointed assistant professor of mathematics at Princeton.

Professor R. L. Wilder, of the University of Michigan, has been granted leave of absence for the coming academic year, for research at the Institute for Advanced Study.

The following appointments to instructorships in mathematics are announced:

Harvard University: Mr. Douglas P. Adams; Mr. Donald H. Ballou; Dr. Albert E. Currier (Benjamin Peirce Instructor); Mr. John H. Curtiss; Mr. Thomas L. Downs, Jr.; Mr. Walter Leighton, Jr.; Mr. Zehman I. Mosesson; Mr. Arthur Sard; Dr. Wladimir Seidel (Benjamin Peirce Instructor); Mr. Frederick H. Steen; Mr. George B. VanSchaack; Dr. Hassler Whitney.

Princeton University: Dr. E. J. McShane; Dr. A. W. Tucker.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

192. Dr. W. I. Miller: *Fundamental regions for the simple group of order 168 in S_4.*

Fundamental regions have been determined for certain groups in S_4 (Price, American Journal of Mathematics, vol. 40 (1918), pp. 108–112). In this paper fundamental regions for the ternary G_{168} are obtained by the use of 21 forms of degree four in which every term is of degree two in the variables x_1, x_2, x_3, and of degree two in the conjugate imaginary variables. These forms are written as the differences of seven positive definite forms, so that their behavior under the group may be studied by means of the permutation group of degree seven. If we set $x_1/x_3=x+iu$, $x_2/x_3=y+iv$, each of these forms, equated to zero, represents a hypersurface in S_4. These hypersurfaces divide the S_4 into 7! regions, each of which has 168 conjugates under the group. A fundamental region is obtained by selecting one region from each of the 30 sets of conjugate regions. Points on the hypersurfaces are also considered. (Received May 15, 1933.)

193. Professor H. S. Vandiver: *On algebraic rings and Abelian groups.*

The methods employed in this article depend on certain representations of the elements of a finite Abelian group (in particular what is called the p-adic representation) which lead also to two possibly new proofs of the existence of a basis for this type of group. Very simple derivations of the known main results and some new results are obtained in the theories of finite fields and rings. (Received May 19, 1933.)

194. Professor L. E. Dickson: *Every large number is a sum of nine values of a cubic polynomial in x.*

The author gives cases of universal theorems described by the title (Received May 4, 1933.)

195. Professor B. A. Bernstein: *A set of four postulates for Boolean algebra in terms of the "implicative" operation.*

The author obtains, as the main object of his paper, a set of four independent postulates for Boolean algebra expressed in terms of the operation

"$\supset$," the "relation of implication" of *Principia Mathematica*. The number of postulates is thus considerably smaller than the number (eight, including an inadvertently omitted existence postulate) in Huntington's set of postulates expressed in terms of "$\supset$" (Proceedings of the National Academy of Sciences, vol. 18(1932), p. 180). The author also shows that there is a very close relation between the operation "$\supset$" and the operation "$-$" of "exception," by virtue of which relation any set of postulates in terms of "$-$" is essentially also a set of postulates in terms of "$\supset$," and vice versa. Finally, the author derives from his postulates the "theory of deduction" of *Principia Mathematica*. This derivation brings out the fact, perhaps more directly than has been done before, that the propositions of the theory of deduction are all propositions in the general logic of classes. (Received May 16, 1933.)

196. Mr. A. M. Tuttle: *A system of independent mathematical postulates for Dirac's quantum mechanics.*

This paper exhibits a system of postulates which are sufficient to furnish a rigorous formal foundation for a type of mathematics used by Dirac in his quantum mechanics (Dirac, *Principles of Quantum Mechanics*, Oxford, 1930). The first twelve postulates furnish a calculus for the system. The notion of a general linear process has been introduced instead of the processes of summation and integration which Dirac uses. In the first twelve postulates this process is not so restricted as to exclude an ordinary integration and many of Dirac's results are obtained upon this basis. However, in order to derive all of Dirac's expansion theorems, it has been necessary to add two more postulates which so restrict the general linear process as to exclude an ordinary integration. The postulates are shown to be consistent and independent. (Received May 19, 1933.)

197. Professor H. S. Vandiver: *Summary of results and proofs on Fermat's last theoem.* Seventh paper.

The main result obtained is as follows. If l is an odd prime and $x^l+y^l+z^l=0$ is satisfied in rational integers, none zero, and prime to the odd prime l, then the second factor of the class number of the algebraic field defined by $e^{2i\pi/l}$ is divisible by l. Other criteria are also found and the proofs indicated. (Received May 19, 1933.)

198. Professor A. A. Albert: *Integral domains of rational generalized quaternion algebras.*

In recent years several papers on the integers of generalized quaternion algebras over R have appeared and various special cases have been considered. But the general case has not yet been completed; in fact the question as to whether it would not be possible to eliminate many of these special cases has not been discussed. The present paper utilizes simple algebraic transformations as well as the theory of ternary quadratic null forms to obtain a remarkable canonical form for any generalized quaternion algebra over R. Moreover the author shows that for algebras in the canonical form there is a single domain of integrity containing the basal units, and determines this domain. (Received May 20, 1933.)

199. Professor H. L. Olson: *Bilinear correspondences in two and in three dimensions.*

Every bilinear form in $n+n$ variables $x_1, \cdots, x_n, y_1, \cdots, y_n$ can be interpreted geometrically as determining a correspondence, in terms of homogeneous coordinates, between points and hyper-planes (linear subspaces of $n-2$ dimensions) in a flat space of $n-1$ dimensions. This paper classifies, projectively, such correspondences in $3+3$ and $4+4$ variables, and presents a canonical form for each class. (Received May 17, 1933.)

200. Miss A. V. Newton: *Consecutive covariant configurations at a point of a space curve.*

The purpose of this paper is to make some contributions to the projective differential geometry of space curves. There are many configurations covariantly connected with a point P of such a curve, such as tangent line, osculating cone, and Halphen point. As the point P moves along the curve each covariant point describes a curve, each covariant line describes a surface, and each covariant surface has an envelope. On considering a point Q infinitesimally near to the point P and setting up a transformation of coordinates between local coordinate systems at P and Q, it becomes possible to obtain considerable information about these curves and envelopes. In particular, one can find the tangent lines of the curves, the characteristics of the envelopes, and the foci of the edges of regression of the envelopes. (Received May 20, 1933.)

201. Professor A. H. Copeland: *Consistency of the conditions determining Kollektivs.*

A rigorous basis for the statistical theory of probability is obtained by the establishment of the consistency of the conditions determining the set of *Kollektivs*. The author shows that, in a certain sense, the physical situation must be in agreement with this theory. In this respect the theory differs from all other physical theories. Moreover, the *Kollektiv* establishes a closer relationship between physical situations and their mathematical formulations. The conditions determining *Kollektivs* are due to von Mises. Consistency can be established by means of the construction of sequences of points satisfying the conditions. The consistency of the conditions for *Kollektivs* of a very special type can be shown by means of the *nombres normaux* of Borel. For this type the conditions assume a much simpler form. In this paper it is shown that a slight modification of the conditions for the more general type is essential to achieve consistency. It is further shown by a method similar to that of Borel that these modified conditions can be satisfied. (Received May 17, 1933.)

202. Mr. T. N. E. Greville: *Invariance of the property of admissibility under certain general types of transformations.*

This paper is concerned with a set of transformations on certain sequences, which include the type of *Kollektiv* called by von Mises "*die einfachste alternative*" and the admissible number of Copeland. It is possible to formulate completely in terms of the transformations of this set a large class of problems aris-

ing in the classical theory of probability for which the operations given by von Mises are not adequate. The consistency of the assumptions of the classical theory is investigated by means of certain properties of the sequences which are invariant under these transformations. The method used in demonstrating the invariance of the properties in question is similar to that employed by Borel in proving the existence of *nombres normaux*. (Received May 17, 1933.)

203. Dr. Francis Regan: *The application of the theory of admissible numbers to time series with variable probability.*

A time series is a sequence of occurrences which may be represented by a set of points on a time axis. In this paper the author is interested in the time series whose points are determined with relation to a distribution function which assigns a definite probability to every interval of the time axis. The probability varies according to the length and position of the interval. The concept of admissibility is extended to this type of time series by setting up, in accordance with the above distribution function, an imaginary set of points on the time axis. The statistical point of view of probability is applied to this series and the set is required to satisfy the fundamental assumptions of the theory of probability. Thus the sequences of successes and failures must be represented by the digits of admissible numbers. For every interval a different set of conditions is obtained and hence the number of conditions imposed has the power of the continuum. The author shows how to obtain a set of points for which these conditions are satisfied. (Received May 17, 1933.)

204. Professor H. M. Gehman: *Points of connectivity.*

A point of connectivity of a set M is a point p of M such that every connected subset of M containing p is a non-cut set of M. It is shown that there are exactly ten types of sets which have points of connectivity. The proofs are based on the set of three axioms mentioned in a previous abstract (No. 39–5–152). The results of this paper generalize theorems due to Zarankiewicz (Fundamenta Mathematicae, vol. 5 (1924), pp. 11–13). (Received May 19, 1933.)

205. Dr. N. E. Rutt: *Some theorems on triodic continua.*

Concerning any compact plane continuum K, it is proved in this paper that if K does not divide the plane and is, for any point k of it, expressible as the sum of two of its subcontinua neither of which conains K and both of which contain k, then K is triodic; and that if K, for any point k of it, is the sum of three of its subcontinua each of which contains k and none of which is contained in the sum of the other two, then K contains a triod. Concerning any collection of non-compact plane continua it is proved that if each is the sum of two of its non-compact proper subcontinua, then the elements of the collection are not both mutually exclusive and uncountable. (Received May 16, 1933.)

206. Mr. C. H. Wheeler, III: *A type of homogeneity for continuous curves.*

The author investigates in this paper conditions under which a compact, locally connected continuum M may be cyclic element homogeneous, i.e., given any two non-degenerate cyclic elements of M, there exists a homeomorphism which sends M into itself in such a way that one of these elements is sent into the other. The case where there are only a finite number of non-degenerate cyclic elements is completely treated. In addition, the investigation has been extended to some cases where the number of such elements is infinite. (Received May 18, 1933.)

207. Mr. R. R. Lyle: *Centers and axes of symmetry.*

The idea of a set of points in the euclidean plane being symmetrical to a center or to an axis is not new. As far as the author knows, no one has previously studied the relations between centers and axes of symmetry for sets having more than one center and more than two axes. This paper contains numerous theorems concerning relations between the number of centers and axes of symmetry of a given set. Examples are given showing the existence of sets having the properties stated in the theorems. (Received May 19, 1933.)

208. Mr. B. B. Sharpe: *Arcs of symmetry.*

We shall say that the arc AB is an arc of symmetry of a set M if $M-(M\cdot AB)$ can be expressed as the sum of two mutually separated sets L and R, such that there exists a homeomorphism T having the following properties: (1) $T(M+AB)=M+AB$; (2) if $X=AB$, then $T(X)=X$; and (3) $T(L)=R$. It is shown that an arc of symmetry is an invariant of the homeomorphic geometry of any space, and that if a homeomorphism T exists, then there is a homeomorphism U having properties (1), (2), and (3) and also such that $U^2(X)=X$ for each point of $M+AB$. Properties of sets which have an arc of symmetry and also sufficient conditions for a set to have an arc of symmetry are considered. The following theorem is proved: If M can be expressed as the sum of a finite number of simple closed curves $C_1, C_2, \cdots, C_n$, each pair of which have a set K in common such that for $i, j=1, 2, 3, \cdots, n$, the points of K occur on C_i in the same cyclic order as they occur on C_j, then M has an uncountably infinite number of arcs of symmetry. (Received May 19, 1933.)

209. Professor J. W. Campbell: *The clock problem in relativity.* Second paper.

The writer's first paper on this subject appeared in the January, 1933, number of the Philosophical Magazine. The device of using a fictitious gravitational potential was there used, and the solution obtained is applicable to paths of any shape and described in any manner, provided only that the speed is not too great. The present paper makes use of rotating axes, and the solution is applicable to motion on a straight line which is not in the vicinity of ponderable matter, though without any restriction as to the magnitude of the speed. Moreover, for the case treated, there is a more vivid picture obtained of the phenomenon of time distortion than is obtained by using a fictitious gravitational potential. (Received May 15, 1933.)

210. Professor Frederick Creedy: *A hyperbolic vector diagram and symbolic method.*

The paper relates to the application of a two dimensional hyperbolic geometry to the study of electrical transients of the form $Ae^{at}+Be^{bt}$. It introduces a geometrical operator k which changes a vector whose components are (a, b) into another whose components are (b, a). Thus $k(a, b)=(b, a)$ and $k^2=1$. Extranormal numbers of the form $x+ky$ are defined and an extranormal number plane corresponding to the complex number plane (but with the relations of hyperbolic instead of euclidean geometry) studied. In terms of this system a transient may be completely represented by Ae^{pt} (A and p being extranormals). Geometrical constructions derived from hyperbolic geometry are developed for the study of various problems relating to electrical transients. The scalar product of two vectors $a+kb$ and $c+kd$ is given a double definition in the plane, one serving for euclidean and the other for hyperbolic geometry, so that the same formulas are capable of two geometrical interpretations, one in each system. (Received May 19, 1933.)

211. Dr. P. G. Hoel: *Certain problems in the theory of closest approximation.*

This paper is concerned with properties of minimizing sums in the field of approximation theory of least mth powers for values of m less than one. With suitable restrictions it is shown that the orthogonality property of the error function holds; and, with the introduction of root multiplicity bounds as a generalization of root multiplicities, that certain continuous error functions possess at least a fixed total number of roots and root multiplicity bounds. Under rather light restrictions it is shown that the coefficients of a minimizing sum are continuous functions of m for all positive values of m. (Received May 18, 1933.)

212. Dr. W. T. Reid: *Discontinuous solutions of the non-parametric problem of Mayer in the calculus of variations.*

The question of discontinuous solutions in the simple problem of the calculus of variations in parametric form has been treated rather thoroughly (See L. M. Graves, this Bulletin, vol. 36 (1930), pp. 831–846), whereas for the non-parametric problem relatively few results have been obtained. This paper treats discontinuous solutions for the general problem of Mayer in non-parametric form, and in terms of the characteristic numbers of a boundary value problem associated with the second variation there is given an effective extension of the Jacobi condition. Sufficient conditions for a minimum are then established by the method used in the case of extremal arcs by Bliss and Hestenes (Transactions of this Society, vol. 35 (1933), pp. 305–326 and pp. 479–490). For the accessory boundary value problem in terms of which the analogue of Jacobi's condition is phrased the boundary conditions apply at more than two points, but it is significant that this problem may be transformed into one for which the boundary conditions apply at only two points, and of the type recently considered by the author (American Journal of Mathematics, vol. 54 (1932), pp. 769–790). (Received May 19, 1933.)

213. Dr. M. R. Hestenes: *Sufficient conditions for the problem of Bolza in the calculus of variations.*

Sufficient conditions for the problem of Bolza have been given by Morse (American Journal of Mathematics, vol. 53 (1931), pp. 517–546) and Bliss (Annals of Mathematics, vol. 33 (1932), pp. 261–274) for arcs which are not only normal relative to the end conditions but also normal on every sub-interval. The latter normality condition prevents these sufficiency conditions from being applicable without further modification to the general problem of Mayer and to the problems of the various classes of Caratheodory (Commentarii Mathematici Helvetici, vol. 5 (1933), pp. 1–19). In the present paper a set of sufficient conditions for the problem of Bolza is given assuming only normality relative to the end conditions. No assumptions regarding normality on the sub-intervals are needed. The conditions here given are therefore applicable at once to the problems of Mayer and of Caratheodory. This result is obtained by replacing the usual Mayer condition by a new condition involving a bilinear form with certain linear conditions adjoined. This new condition is a consequence of the theory of broken extremals applied to the problem of the second variation. The results of this paper are extensions of those recently announced by the author for the problem of Lagrange (Abstract No. 39–5–124). (Received May 23, 1933.)

214. Mr. H. L. Turrittin: *Asymptotic solutions of certain ordinary differential equations associated with multiple roots of the characteristic equation.* Preliminary report.

In certain instances the asymptotic nature of a fundamental set of solutions of an ordinary linear differential equation containing a parameter ρ has been ascertained in the neighborhood of $\rho = \infty$. The present paper contains the first results obtained by the author in attempting to extend Birkhoff's analysis (Transactions of this Society, vol. 9 (1908), pp. 219–231) to include certain cases in which the characteristic equation possesses multiple roots; subject to the proviso that if any two roots are equal, they are identically equal throughout the entire interval under consideration. The asymptotic nature of a fundamental set of solutions is deduced subject to the assumption that the actual determination of the formal solutions of the type described by Tamarkin (Master's Thesis, Petrograd, 1917) involves at most the solution of first, and not higher, order differential equations. (Received May 19, 1933.)

215. Professor I. A. Barnett: *Functional invariants of integro-differential equations.*

In a previous paper the author pointed out how the study of the invariants and covariants of a system of linear differential equations of the second order could be extended to the analogous consideration of the functional invariants and covariants of an integro-differential equation of the second order. In this paper an existence theorem concerning the solutions of such equations is considered. It is shown further that there is an infinite system of functional invariants involving derivatives of all orders. These invariants turn out to be

the traces of certain kernels arising from the coefficients of the original equation. Finally, the author suggests an application of these results to the study of projective properties of surfaces in a certain type of function space. (Received May 19, 1933.)

216. Dr. E. W. Titt (National Research Fellow): *Cauchy's problem for systems of second order partial differential equations.*

By a method similar to that used by E. Cartan (Bulletin de la Société Mathématique de France, vol. 59 (1931)) on systems of first order equations, the author obtains a canonical form for systems of second order partial differential equations, linear in the second derivatives. New considerations enter into the second order problem; for example, in the first order case there is only one possibility for a canonical form while in the second order case it is not evident which of two possibilities will give the desired results. Use of Riquier's theory of orthonomic systems leads to a clear statement of Cauchy's problem for the systems under consideration. The characteristic surfaces are those over which the results on Cauchy's problem fail to apply. The theory applies to Einstein's gravitational equations for free space. A theorem which shows that Cauchy's problem for a characteristic fails by becoming indeterminate is proved using the theory of normal systems of differential equations of Thomas and Titt (Annals of Mathematics, ser. 2, vol. 34 (1932)), an extension of Riquier's work. For arbitrary coordinate systems the present theory handles the second order invariantive systems treated by Thomas without using the theory of differential invariants of generalized spaces and without reducing the system to an equivalent system of first order equations. (Received May 22, 1933.)

217. Mr. H. H. Alden: *Properties of differential equations arising from properties of the solution curves.*

F. R. Bamforth and the author have shown (this Bulletin, Abstract No. 39–1–7) that $dy/dx=f(x, y)$ may have a unique solution extending from boundary to boundary through each point of the region of definition of $f(x, y)$ without $f(x, y)$ being continuous or satisfying any of a number of well known uniqueness conditions. In the present note it is shown that the hypotheses usually placed on $f(x, y)$ to insure that the solutions have such further properties as differentiability with respect to initial conditions and analyticity are actually necessary for the solutions to have these properties. (Received, May 13, 1933.)

218. Professor O. D. Kellogg: *Converses of Gauss' theorem on the arithmetic mean.*

Let T be a bounded domain of the plane. Let the function $u(p)$ be continuous in T. Let $c(p)$ denote a circle about p as center, and let $A(u, c)$ denote the arithmetic mean of u on c. Then, according to Koebe's converse of Gauss' theorem, u is harmonic in T if, for each point p in T, $u(p)=A\{u, c(p)\}$ for every circle $c(p)$ contained in T. The question arises as to the properties of u if one supposes only that, for each p of T, $u(p)=A\{u, c(p)\}$ for some one circle

contained in T, or, more generally, if one supposes only that $u(p)$ is equal, at each p of T, to the solution of the Dirichlet problem, with boundary values u, for some normal domain containing p and strictly contained in T. In this paper results are obtained in regard to a function u of this character. These results are concerned chiefly with the bounds of u, and conditions on u in the neighborhood of the boundary of T that imply that u is harmonic in T. The reasoning is valid in space, and holds, under suitable restrictions on u, even when the boundaries of the normal domains have points in common with the boundary of T. The results were obtained by the late Professor Kellogg. They were prepared for publication by J. J. Gergen. (Received May 13, 1933.)

219. Dr. Hassler Whitney (National Research Fellow): *Derivatives, difference quotients, and Taylor's formula.*

Let $f(x)$ be measurable in the closed interval I. A necessary and sufficient condition that $f(x)$ be a polynomial of degree at most $m-1$ is that the mth difference quotient $\Delta_h^m f(x)=\sum_{i=0}^{m}(-1)^{m-i}\binom{m}{i}f(x+ih)/h^m\to 0$ uniformly as $h\to 0$. A necessary and sufficient condition that $f(x)$ have a continuous mth derivative is that there exist a continuous function $f_m(x)$ in I such that $\Delta_h^m f(x)\to f_m(x)$ uniformly as $h\to 0$, or, that there exist functions $f_0(x)=f(x)$, $f_1(x), \cdots, f_m(x)$ in I such that $f(x+h)=\sum_{i=0}^{m}f_i(x)h^i/i!+R(x, h)$, where $R(x, h)/h^m\to 0$ uniformly as $h\to 0$, then $d^i f(x)/dx^i=f_i(x)$, $(i=1, \cdots, m)$. (Received May 13, 1933.)

220. Professor J. F. Ritt and Dr. J. L. Doob (National Research Fellow): *Systems of algebraic difference equations.*

A definition is given of *irreducible* systems of algebraic difference equations. It is shown that every system of such equations is equivalent to a finite set of irreducible systems. (Received May 17, 1933.)

221. Dr. W. J. Trjitzinsky: *The general case of non-homogeneous linear differential equations.*

In the present paper the author considers the asymptotic nature of solutions, in the complex plane and in the vicinity of a singular point, of equations of the type indicated in the title. The treatment is given for the general case of *unrestricted* roots of the corresponding characteristic equations. This treatment is based on another work of the author in which the analytic theory of the analogous problem for homogeneous differential equations is developed. (Received May 20, 1933.)

222. Professor J. A. Shohat and Dr. Clement Winston: *On mechanical quadratures.*

This paper is an extension of one on the same subject presented to the Society at its April, 1933, meeting in New York by Dr. Winston. It gives a further discussion of the coefficients $H_{i,n}$ in the mechanical quadratures formula $\int_a^b f(x)p(x)dx=\sum_{i=1}^{n}H_{i,n}f(x_{i,n})+R_n(f)$, $(H_{i,n}=\int_a^b(\phi_n(x)p(x)dx)/((x-x_{i,n})\cdot\phi_n'(x_{i,n}))$, $\int_a^b\phi_n(x)\phi_m(x)p(x)dx=\delta_{m,n})$ for the classical orthogonal polynomials,

and especially for those of Hermite. Upper and lower bounds for the coefficients $H_{i,n}$ are obtained using their extremal properties and certain expressions involving $K_n(x) \equiv \sum_{i=1}^{n} \phi_i^2(x)$; also a differential equation of the first order for $K_n(x)$ is derived. Special considerations in the Hermite case lead to some interesting asymptotic relations, as, for example, $\lim_{n\to\infty} K_n(x)/(2n)^{1/2} = e^{x^2}/\pi$, ($|x| < (2n - cn^{1/3})^{1/2}$, c constant). (Received May 20, 1933.)

223. Dr. M. R. Hestenes: *A note on the Jacobi condition for parametric problems in the calculus of variations.*

The present note gives a simple modification of the treatment of the Jacobi condition given by Bliss (Transactions of this Society, vol. 17 (1916), pp. 195–206). Instead of defining conjugate points in terms of solutions η_i of the Jacobi equations which satisfy the conditions $y_i'\eta_i = 0$, as was done by Bliss, they are defined in terms of solutions η_i which satisfy the conditions $y_i'\eta_i' = \text{constant}$. With the help of these solutions the usual criteria for conjugate points are deduced. Since there are but $2n-2$ linearly independent solutions of the first type and $2n$ of the second type, the treatment here given seems to be simpler and more symmetric than that of Bliss and is almost identical with that usually given for the non-parametric problems. (Received May 23, 1933.)

224. Professor D. N. Lehmer: *Franklin magic squares.*

In a letter to Peter Collinson, Benjamin Franklin describes a method of constructing magic squares of even order, illustrating it with two squares, one of order 8 and one of order 16. These squares are magic, not only in the rows and columns and "broken diagonals" but also *im kleinen*, that is to say; the sum of the elements in any sub-square of order 2 is constant over the whole square. In this paper the author develops what is without doubt the method employed by Franklin in constructing these squares and shows how it may be employed in squares of all even orders, and brings the whole into a unified theory. (Received May 10, 1933.)

225. Professor A. A. Albert: *A generalization of a theorem of Dirichlet.*

In 1840 Dirichlet outlined a proof of the theorem that every properly primitive binary with non-square discriminant represents infinitely many primes. This theorem was later proved by H. Weber, A. Meyer, and F. Mertens, but seems never to have been extended to n-ary or to non-classical forms. In the present paper the author applies the Hilbert Irreducibility Theorem to show that every properly primitive (classical or non-classical) n-ary quadratic form is equivalent to a form $f(x_1, \cdots, x_n)$ whose sub-forms $f_r = f(x_1, \cdots, x_r, 0, \cdots, 0)$ are all properly primitive and non-singular while f_2 has non-square discriminant. The extension of the Dirichlet theorem to non-classical forms is immediate and proves that every n-ary in the above canonical form represents infinitely many primes, in particular those represented by f_2. (Received May 20, 1933.)

226. Professor C. G. Latimer: *Note on the invariants of the class group of a cyclic field.*

Let F be an algebraic field which is cyclic and of odd degree e, with respect to the rational field. The classes of ideals in F are the elements of an Abelian group G, which is characterized by certain prime power invariants. Let the distinct fields defined by the various e-th roots of unity $\neq 1$ be $K_1, K_2, \cdots, K_n$. The purpose of this note is to prove the following theorem. Let p be a rational prime and k a positive integer. If the prime ideal divisors of p in $K_1, K_2, \cdots, K_n$ are of degree $g_1, g_2, \cdots, g_n$ respectively, the number of invariants of G which are equal to p^k may be written in the form $g_1x_1+g_2x_2+\cdots+g_nx_n$ where each $x \geqq 0$. Since the class number h, of F, is the product of the invariants, it follows that h is the product of the norms of n ideals, one in each of the fields K_i. For the case where e is a prime, this result was previously obtained by the writer in a different manner. (Transactions of this Society, vol. 35 (1933), p. 417.) (Received May 19, 1933.)

227. Dr. H. P. Thielman: *Note on Riemannian function space geometry.*

In this note the following result is obtained: *Every Riemannian function space can be immersed in a euclidean function space.* A Riemannian function space is here defined in accordance with the definition found in the literature (A. D. Michal, American Journal of Mathematics, vol. 50, p. 516) as one in which the element of length is given by $(ds)^2=\int_a^b\int_a^b g(\alpha, \beta)\partial y(\alpha)\partial y(\beta)d\alpha d\beta+\int_a^b g(\alpha)\partial^2 y(\alpha)d\alpha>0$ where $g(\alpha, \beta)=g(\beta, \alpha)$, $g(\alpha)\neq 0$ in (a, b), and $\partial y(\alpha)$ is continuous and does not vanish identically in (a, b). If $g(\alpha, \beta)\equiv 0$, $g(\alpha)\equiv 1$ the space is a euclidean function space. The equivalence is established under the group of linear functional transformations of the third kind: $g(s)=K(s)\bar{y}(s)+\int_a^b K(s, \alpha)\bar{y}(\alpha)d\alpha$, $K(s)\neq 0$ in (a, b). (Received May 23, 1933.)

228. Dr. A. E. Ross (National Research Fellow): *Positive quaternary quadratic forms representing all but a finite number of integers.*

Ramanujan, having determined all universal positive quaternary quadratic forms of the form $ax^2+by^2+cz^2+du^2$, proposed the problem of determination of all such forms representing all but a finite number of integers (Collected Papers, Cambridge University Press (1927), p. 169). This problem was later solved by H. P. Kloosterman (Acta Mathematica, vol. 49 (1926), pp. 407–64). In the present paper, the author proposes to study positive quaternary quadratic forms representing all but a finite number of integers, without imposing the restriction that these forms should contain no cross-product terms. (Received May 23, 1933.)

229. Dr. B. F. Kimball: *The generalized Bernoulli polynomial and its relation to the Riemann zeta function.*

The generalized Bernoulli polynomial $B_s(x)$ is defined as the solution of the difference equation $B_s(x+1)-B_s(x)=sx^{s-1}$, x real and positive, s a complex

variable; this solution to be analytic in s and such that $\lim_{x\to\infty} B_s(x)=0$, $R(s)<0$. It is found that under the above conditions $B_s(x)$ is an entire function of s, that it is unique, and that it reduces to the classical Bernoulli polynomials when s is a positive integer or zero. It is found that the Riemann zeta function is related to it by the equation $(-s)\zeta(1-s, x)=B_s(x)$ for all values of s, x real and positive. Thus $-\zeta(-s, x)$ is the unique solution of the difference equation $f(s, x+1)-f(s, x)=x^s$, $s\neq -1$, x real and positive, which is analytic in s except at $s=-1$, and such that $\lim_{x\to\infty} f(s, x)=0$, $R(s)<-1$. (Received June 5, 1933.)

230. Dr. D. C. Duncan: *The completely symmetric rational self-dual septimic.*

A discussion of the completely symmetric elliptic self-dual septimic will appear shortly in this Bulletin. The present paper treats the completely symmetric rational septimic, having five of each of the simple singularities, cusps, inflexions, crunodes, acnodes isolated and ordinary double-tangents. The ten collineations and ten correlations under which the curve is invariant are listed. Of the correlations five are polarities by rectangular hyperbolas and one a polarity by an imaginary circle touching the hyperbolas at the ends of their conjugate axes. (Received June 5, 1933.)

231. Dr. R. E. A. C. Paley: *A special integral function.*

In this paper the late R. E. A. C. Paley gives an example of an entire function which for the minimum modulus $m(r)$ is bounded, but which possesses no asymptotic path. The "segments" of the series defining the non-differentiable Weierstrassian function play essential roles in this construction. (Received June 5, 1933.)

232. Mr. T. P. Palmer: *Postulates for logic.*

The following set of independent postulates suggested by papers of H. M. Sheffer, N. G. P. Nicod, and C. E. Van Horn, is proved equivalent to E. V. Huntington's postulates for the *informal Principia*. The base is a class, T, a class, F, and an operation, $|$, (stroke). 1. If p in T and q in T then $p|q$ in F. 2. If p in T and q in F then $p|q$ in T. 3. If p in F and q in T then $p|q$ in T. 4. If p in F and q in F then $p|q$ in T. 5. If p in T then p not in F. It is shown that if, in the *formal Principia*, any one proposition is both true and false, then every proposition is both true and false and the *formal Principia* becomes trivial. Thus the addition of postulate 5 above to *formal Principia* is not abhorrent. The postulates (except 5) are proved from *formal Principia*, and, with K suitably defined, *formal Principia* is proved from these. The above postulates are generalized to give a system with multiple truth values, K defined, and *Principia's* postulates are proved for this system. (Received June 8, 1933.)

233. Professor G. A. Miller: *Groups in which every operator has at most a prime number of conjugates.*

The author proves that if every non-invariant operator of a group has a given prime number of conjugates under this group then the order of the

commutator subgroup of this group is this prime number and its operators are invariant under the group. It is also proved that if a group contains more than one Sylow subgroup of order p^m then it contains a set of conjugate operators whose order is a power of p and whose number exceeds p. (Received June 14, 1933.)

234. Professor A. J. Maria: *Concerning the equilibrium point of Green's function for an annulus.*

Let r and r_0 be the distance from the center of the two concentric circles of the equilibrium point and pole, respectively. The points lie on the same diameter. It is shown that $dr/dr_0>0$ for any position of the pole interior to the ring on a fixed diameter, and that $\lim dr/dr_0=0$ as r_0 approaches the inner or outer boundaries of the ring. A formula is given for the maximum displacement of r from the geometric mean circle. (Received June 15, 1933.)

235. Professor E. V. Huntington: *A second set of independent postulates for the informal Principia Mathematica.*

The set given in Appendix II of the author's paper in the Transactions of this Society, vol. 35 (1933), pp. 274–304, 557–558, contains a redundancy. Example 5 (page 304) is erroneous, and Postulate 5 (page 301) is deducible from Postulates 1, 2, 3, 4, 6, 7. A second set of postulates for the same system is the following. *Base*: $(K, T, +, ')$, where T is a proper subclass in K, that is, K is dichotomized into two non-empty, non-overlapping subclasses T and F. Postulates: 1°. If a and b are in K, then $a+b$ is in K. 2°. If a is in K, then a' is in K. 3°. If b is in T, then $a+b$ is in T. 4°. If b is in T, then $b+a$ is in T. 5°. If a is in F, then a' is in T. 6°. If a is in T, then a' is in F. 7°. If $a+b$ is in T and a is in F, then b is in T. In this set no postulate involves more than two elements. A similar set is obtained by interchanging $+$ and $\times$, and T and F. (Received June 16, 1933.)

236. Professor I. J. Schwatt: *The sum of like powers of a series of numbers forming an arithmetical progression and the Bernoulli numbers.*

Since the beginning of the eighteenth century mathematicians, in expanding $\sum_{k=1}^{n-1}(\pm 1)^{k-1}k^p$ into an explicit function in powers of n, have always assumed the coefficients of the expansion to contain a Bernoulli number in order, that is, the Bernoulli numbers are assumed to be known. The author has succeeded in devising methods which render a direct expansion of the function and in this way numerical expressions for the Bernoulli numbers. By means of this method the general term of the expansion, and therefore the general Bernoulli number, can be found. This was impossible by the method used heretofore. (Received June 17, 1933.)

237. Dr. J. W. Bower: *The problem of Lagrange with finite side conditions.*

In this paper two methods are used for the development of a complete theory of solution of the problem of Lagrange with finite side conditions.

The first transforms it by differentiation of the side conditions and reduction of the end conditions into a problem of Lagrange with differential equations as side conditions and first end-point variable. Admissible arcs for the new problem are normal relative to the end conditions and possess normality on every subinterval relative to similarly reduced end conditions but not with respect to fixed end-points as is required in the theories hitherto given for problems with variable end-points. The second method replaces the original problem by an equivalent simple problem of the calculus of variations in xz-space. This is done by adjunction of suitably chosen auxiliary equations to the side conditions and solution of the resulting system for the variables y in terms of new variables z. The well-known necessary and sufficient conditions for a minimum for this simple problem are interpreted for the original problem in xy-space, and the auxiliary functions are eliminated from the resulting conditions. Identical theories of solution of the original problem are deduced by application of these two methods. (Received June 19, 1933.)

238. Mr. E. N. Oberg: *The approximate solution of integral equations.*

The aim of this paper is to investigate the approximate representation of the solution of a given Fredholm linear integral equation $L(u)=f(x)$ by means of a polynomial. The criterion of approximation is the minimizing of the integral of $|f(x)-L(P_n)|^m$, where $P_n(x)$ is a polynomial of the nth degree, and the principal problem is that of the convergence of $P_n(x)$ toward $u(x)$ as n becomes infinite, $u(x)$ being the unique continuous solution of the integral equation. This problem for $m=2$ has been discussed recently by Picone, Krawtchouk, Enskog, and others, but the methods and results are different. The convergence proof for $m \geqq 1$ is based indirectly on Hölder's inequality, and for $m<1$ on an extension of Markoff's theorem on the derivative of a polynomial. (Received June 17, 1933.)

NEW PUBLICATIONS

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, October 28, 1933.

Abstracts must be in the hands of Associate Secretary J. R. Kline, 501 West 116th St., New York City, not later than October 7. Professor R. L. Jeffery will deliver an address on *Theories of integration*. Abstracts received by the Secretaries by September 9 will appear in the September issue of the BULLETIN.

CINCINNATI, OHIO, December 1-2, 1933.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than November 11. Abstracts received by the Secretaries by November 4 will appear in the November issue of the BULLETIN.

PASADENA, CALIFORNIA, December 2, 1933.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, Calif., not later than November 11. Abstracts received by the Secretaries by November 4 will appear in the November issue of the BULLETIN.

CAMBRIDGE, MASSACHUSETTS, ANNUAL MEETING, December 27-29, 1933.

This meeting is to be held in conjunction with the A.A.A.S. and the M.A.A. There will be a symposium on *General Analysis*. The Bôcher Prize will be awarded. Abstracts must be in the hands of the Secretary of the Society, 501 West 116th St., New York City, not later than November 30, 1933. Abstracts received by the Secretaries by November 4 will be published in the November issue of the BULLETIN.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1932, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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